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Allama Iqbal Open University

Department of Mathematics

Mathematics-II (248)

Level: Matric

Semester: Autumn 2026

Assignment No. 2 (Units 6 to 9)

Total Marks: 100 Pass Marks: 40

SOLVED ASSIGNMENT GUIDE — For Study & Reference

Note for Students: This guide is for study and reference only. Read each model answer, understand the method, and then write the assignment in your own words and your own hand. Copying any guide word for word is against the University's plagiarism warning printed in the question booklet, so always rewrite the working in your own style after you have understood it.

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Question 1 (20 Marks)

Question 1: (a) A belt and a wallet cost Rs. 42, while 7 belts and 4 wallets cost Rs. 213. Calculate the cost of each item. (b) Given $A = [[1, -1], [2, 3]]$, verify that $A^2 - 4A + 5I = 0$. (20)

Introduction

This question pairs the two faces of linear algebra at Matric level. Part (a) is a shopping problem with two unknowns, solved by forming two simultaneous equations; such problems are the oldest and most practical use of algebra, because prices, mixtures, ages and speeds all produce exactly this pattern of two facts about two unknown quantities. Part (b) moves from numbers to matrices:

the student must square a 2 × 2 matrix, combine it with scalar multiples of itself and of the identity matrix, and show that the result is the zero matrix. That statement is not a coincidence — it is an instance of the Cayley–Hamilton theorem, which says that every square matrix satisfies its own characteristic equation — and verifying it by direct computation is excellent practice in matrix multiplication, the operation in which most Matric errors occur.

Part (a): Simultaneous Equations for the Belt and Wallet

Let the cost of one belt be Rs. b and the cost of one wallet be Rs. w. The first sentence gives b + w = 42. The second gives 7b + 4w = 213. Solve by elimination: multiply the first equation by 4 to get 4b + 4w = 168, and subtract it from the second equation; the w terms cancel and 3b = 213 − 168 = 45, so b = 15. Substituting back into b + w = 42 gives w = 42 − 15 = 27. So one belt costs Rs. 15 and one wallet costs Rs. 27. Verification against the second, un-used-in-full condition: 7 belts cost 7 × 15 = Rs. 105 and 4 wallets cost 4 × 27 = Rs. 108, and 105 + 108 = 213, exactly the stated total, so both conditions are satisfied. The substitution method reaches the same answer: from the first equation w = 42 − b, and putting this in the second gives 7b + 4(42 − b) = 213, that is 3b + 168 = 213 and again b = 15. Whichever method is chosen, the discipline is the same — define the unknowns in words, translate each sentence into one equation, solve, and check the answer in the sentence that was not used to find it. Note also that the answer is realistic: both prices are positive whole numbers of rupees, as prices in such textbook problems should be; a negative or fractional answer would signal a translation error.

Part (b): Verifying $A^2 - 4A + 5I = 0$

Let A = [[1, −1], [2, 3]]. First compute $A^2 = A \times A$ row-by-column. The (1,1) entry is (1)(1) + (−1)(2) = 1 − 2 = −1. The (1,2) entry is (1)(−1) + (−1)(3) = −1 − 3 = −4. The (2,1) entry is (2)(1) + (3)(2) = 2 + 6 = 8. The (2,2) entry is (2)(−1) + (3)(3) = −2 + 9 = 7. So A^2 = [[−1, −4], [8, 7]].

Next, 4A = [[4, −4], [8, 12]] and 5I = [[5, 0], [0, 5]], where I is the 2 × 2 identity matrix. Now combine entry by entry: $A^2 - 4A + 5I$ = [[−1 − 4 + 5, −4 + 4 + 0], [8 − 8 + 0, 7 − 12 + 5]] = [[0, 0], [0, 0]] = 0, which verifies the required result. The deeper reason, stated here for understanding rather than as part of the verification, is that the characteristic equation of A is λ^2 − (trace A)λ + det A = $\lambda^2 - 4\lambda + 5 = 0$, because the trace of A is 1 + 3 = 4 and its determinant is (1)(3) − (−1)(2) = 5; the Cayley–Hamilton theorem guarantees that A itself satisfies this equation. A practical consequence is that the relation can be rearranged to find the inverse: from $A^2 - 4A + 5I = 0$, multiplying by A^{-1} gives $A - 4I + 5A^{-1} = 0$, so $A^{-1} = (4I - A)/5 = (1/5)$[[3, 1], [−2, 1]], which the student can check gives det-based inverse (1/5)[[3, 1], [−2, 1]] — in agreement.

Object	Value
A	[[1, −1], [2, 3]]
A^2	[[−1, −4], [8, 7]]
−4A	[[−4, 4], [−8, −12]]
5I	[[5, 0], [0, 5]]
$A^2 - 4A + 5I$	[[0, 0], [0, 0]] = 0 ✓

Using the Verified Identity: Powers of A

A verified matrix identity is a tool, not just a result. Because $A^2 = 4A - 5I$, every higher power of A collapses to the form pA + qI. For example, $A^3 = A \times A^2 = A(4A - 5I) = 4A^2 - 5A = 4(4A - 5I) - 5A = 11A - 20I$, and evaluating gives 11A − 20I = [[11 − 20, −11], [22, 33 − 20]] = [[−9, −11], [22, 13]], and direct multiplication $A^2 \times A$ = [[−1, −4], [8, 7]] × [[1, −1], [2, 3]] gives [[−1 − 8, 1 − 12], [8 + 14, −8 + 21]] = [[−9, −11], [22, 13]] — the same matrix, confirming the

shortcut. For part (a), one more presentation point deserves emphasis: the elimination step multiplied the first equation by 4 precisely because the wallet coefficients were 1 and 4; choosing the multiplier that makes one pair of coefficients equal is the whole art of elimination, and the alternative pairing (multiplying by 7 to eliminate belts first) gives 7w − 4w... in full: 7(b + w) = 294 minus (7b + 4w) = 213 yields 3w = 81, so w = 27 directly. Both orders are correct; a student should choose whichever coefficients look kinder, and then — as throughout this guide — verify in both original statements.

Conclusion

The belt costs Rs. 15 and the wallet Rs. 27, verified against both stated totals (15 + 27 = 42 and 105 + 108 = 213). Direct computation gives $A^2 = [[-1, -4], [8, 7]]$, and combining $A^2 - 4A + 5I$ entry by entry produces the zero matrix, so the identity is verified; it holds because A satisfies its own characteristic equation $\lambda^2 - 4\lambda + 5 = 0$.

Question 2 (20 Marks)

Question 2: (a) The sum of two angles is 100°, The supplement of their first angle exceeds the supplement of the second angle by 40°. Find the angles. (b) Solve by the matrix inversion method: 3x − 4y = 7; 5x − 7y = 12. (20)

Introduction

Part (a) tests whether the student truly understands supplementary angles — two angles whose sum is 180° — as opposed to merely remembering the word. The trick of the question is that taking supplements reverses a difference: if one angle is larger, its supplement is smaller by exactly the same amount, and the problem is built on that see-saw. Part (b) requires one named method, matrix inversion: the pair of equations is written as a single matrix equation MX = B, the inverse of the coefficient matrix is found from the determinant and adjoint, and the solution

follows from X = M⁻¹B. As with the factorisation instruction in Assignment No. 1, Question 5, the named method is part of the question, and this answer follows it step by step before checking the result by substitution.

Part (a): The Two Angles and Their Supplements

Let the two angles be α and β, with α + β = 100°. The supplement of the first angle is 180° − α and the supplement of the second is 180° − β. The statement says the supplement of the first exceeds the supplement of the second by 40°, so (180° − α) − (180° − β) = 40°, which simplifies to β − α = 40°. (Notice the reversal: the first angle's supplement is the larger one precisely because the first angle itself is the smaller one.) We now have the pair α + β = 100° and β − α = 40°. Adding the equations eliminates α: 2β = 140°, so β = 70°, and then α = 100° − 70° = 30°. The two angles are 30° and 70°. Verification: their sum is 100°, as required. Their supplements are 180° − 30° = 150° and 180° − 70° = 110°, and 150° − 110° = 40°, exactly the stated excess, with the supplement of the first (30°) angle indeed the larger, as the question describes. Students should be careful to follow the booklet's wording — 'the supplement of their first angle exceeds the supplement of the second' — in the order given; swapping the order would produce β = 30° and α = 70°, whose supplements differ the wrong way round, and only the verification step reveals such a swap.

Part (b): Solution by Matrix Inversion

Write the system 3x − 4y = 7 and 5x − 7y = 12 in matrix form MX = B, where M = [[3, −4], [5, −7]], X = [[x], [y]] and B = [[7], [12]]. The determinant of M is (3)(−7) − (−4)(5) = −21 + 20 = −1. Because the determinant is not zero, the inverse exists and the system has exactly one solution. For a 2 × 2 matrix [[a, b], [c, d]], the inverse is (1/det) × [[d, −b], [−c, a]] — swap the diagonal entries, change the signs of the off-diagonal entries, and divide by the determinant.

Hence $M^{-1} = (1/-1) \times [[-7, 4], [-5, 3]] = [[7, -4], [5, -3]]$. Then $X = M^{-1}B = [[7, -4], [5, -3]] \times [[7], [12]]$: the first entry is $(7)(7) + (-4)(12) = 49 - 48 = 1$, and the second entry is $(5)(7) + (-3)(12) = 35 - 36 = -1$. So $x = 1$ and $y = -1$. Verification in both original equations: $3(1) - 4(-1) = 3 + 4 = 7$ ✓, and $5(1) - 7(-1) = 5 + 7 = 12$ ✓. The negative value of y often worries students, but simultaneous equations about pure numbers (unlike the prices of Question 1) may perfectly well have negative solutions, and the double substitution check settles the matter.

Step	Working	Result
Determinant	$(3)(-7) - (-4)(5)$	$-1 \neq 0$, inverse exists
Inverse M^{-1}	$(1/-1)[[-7, 4], [-5, 3]]$	$[[7, -4], [5, -3]]$
$X = M^{-1}B$	$[[7(7) - 4(12)], [5(7) - 3(12)]]$	$x = 1, y = -1$

Cross-Checks: Cramer's Rule and the See-Saw Principle

The matrix result of part (b) can be confirmed by Cramer's rule, which uses the same determinant $D = -1$. Replace the first column of M by B: $Dx = (7)(-7) - (-4)(12) = -49 + 48 = -1$, so $x = Dx/D = (-1)/(-1) = 1$. Replace the second column: $Dy = (3)(12) - (7)(5) = 36 - 35 = 1$, so $y = Dy/D = 1/(-1) = -1$. Complete agreement — as there must be, since all correct methods solve the same system. For part (a), the see-saw principle generalises usefully: whenever two quantities have a fixed sum, the difference of their supplements is the negative of their own difference; that is why the data 'sum 100°, supplement-excess 40°' translate immediately into 'difference 40°' with the order reversed, and the pair (30°, 70°) follows from halving 100 − 40 and 100 + 40... precisely, the smaller angle is $(100° - 40°)/2 = 30°$ and the larger is $(100° + 40°)/2 = 70°$. Recognising the sum-and-difference pattern turns a two-equation problem into a ten-second

mental calculation, but the examination answer should still show the equations, because the method marks are awarded for the translation, not only for the numbers.

One further habit deserves recording for part (b): before computing any inverse, glance at the determinant. Here det M = −1 has absolute value 1, which is why the inverse [[7, −4], [5, −3]] contains only whole numbers and the final solution comes out in whole numbers — a small mercy students should notice, because with a determinant such as 7 or −13 every entry of the inverse, and usually the solution, would carry that denominator. Noticing the determinant first also tells you in advance whether the system has a unique solution at all: a zero determinant would mean the two lines are parallel or coincident, no inverse exists, and the correct examination answer is to say so rather than to manufacture numbers. In the present question the two lines 3x − 4y = 7 and 5x − 7y = 12 are not parallel (their gradients 3/4 and 5/7 differ), they cross at exactly one point, and the matrix method finds that crossing point as (1, −1).

Conclusion

The two angles are 30° and 70°: they sum to 100°, and their supplements 150° and 110° differ by exactly 40° in the order stated. By matrix inversion, with det M = −1 and M^{-1} = [[7, −4], [5, −3]], the system gives x = 1 and y = −1, confirmed in both original equations.

Question 3 (20 Marks)

Question 3: (a) Draw a tangent to two unequal touching circles. (b) Draw a tangent to a circle from a point on the circumference. (20)

Introduction

This is the construction question of the assignment, and it is marked on the quality and correctness of the drawing as much as on any writing: the student should work in sharp pencil, keep construction arcs visible, and label every centre, point of contact and line. A tangent is a

straight line that touches a circle at exactly one point, called the point of contact, and the one theorem behind both parts is that the radius to the point of contact is perpendicular to the tangent. When two circles touch each other, they share exactly one point, and at that shared point the two circles have one and the same tangent line — the line perpendicular to the line of centres at the touching point. Part (a) is most naturally answered with that common tangent at the point of contact, and the construction below also shows how a direct common tangent is obtained, so that either reading of 'a tangent to two unequal touching circles' is covered. Part (b) is the simpler perpendicular construction at a point of the circle itself. The figures printed with this answer are accurate scale diagrams of the finished constructions; in the student's own submission the figures must be drawn by hand with ruler and compasses, following the steps word for word.

Part (a): A Common Tangent to Two Unequal Touching Circles

Given: two unequal circles, say with centres O_1 and O_2 and radii $r_1 < r_2$, touching each other externally at the point T on the line of centres O_1O_2. Construction of the tangent at the touching point: (1) Draw the two circles with centres O_1 and O_2, choosing the distance O_1O_2 exactly equal to $r_1 + r_2$ so that the circles touch externally; mark the touching point T where the circles meet on O_1O_2. (2) The tangent required is the line through T perpendicular to O_1O_2. To erect that perpendicular with compasses: with centre T and any convenient radius, draw arcs cutting the line O_1O_2 at two points P and Q on either side of T. (3) With centres P and Q and equal radii greater than PT, draw arcs intersecting above the line at R and below it at S. (4) Join RS. The line RS passes through T and is perpendicular to O_1O_2, and it is the common tangent to both circles at T: it touches the first circle only at T and the second circle only at T. Justification: the radius O_1T is perpendicular to RS and the radius O_2T is also perpendicular to RS, since both radii lie on the same straight line O_1O_2; a line perpendicular to the radius at its endpoint on the circle

is a tangent, so RS is tangent to both circles. If a direct common tangent (touching the circles at two different points) is wanted instead, the standard construction is: on the larger circle draw a concentric auxiliary circle of radius $r_2 - r_1$, draw a tangent from O_1 to that auxiliary circle, transfer the direction by parallels, and the resulting line touches the larger circle at one point and the smaller at another; Figure 2 shows such a direct common tangent as well as the tangent at T. The essential examination point is that touching circles have a common tangent at their point of contact, and its construction is just the perpendicular at T.

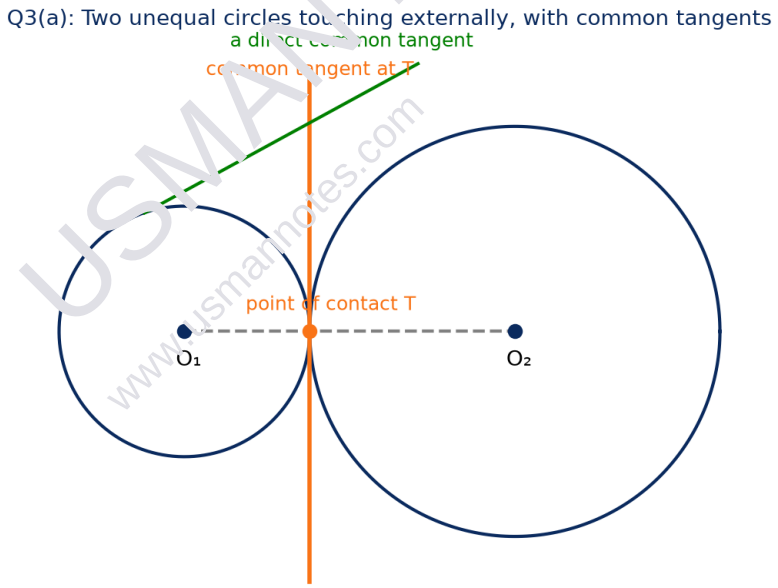


Figure 2: Two unequal circles touching externally at T. The orange line through T, perpendicular to the line of centres, is the common tangent at the point of contact; a direct common tangent is also shown.

Part (b): Tangent from a Point on the Circumference

Given: a circle with centre O and a point P on its circumference; the tangent at P is required. Construction: (1) Draw the circle and join OP, extending the line a little beyond P. (2) With centre P and a convenient radius, draw arcs cutting the line OP produced at two points, one on each side of P. (3) With those two points as centres and equal radii larger than half their

separation, draw arcs intersecting on both sides of OP, at points say C and D. (4) Join CD. The line CD passes through P and is perpendicular to OP, and it is the required tangent, touching the circle only at P. Justification: CD is the perpendicular bisector construction applied at P, so angle O P (tangent) is a right angle; the radius to a point of contact is perpendicular to the tangent, and conversely any line through P perpendicular to OP can meet the circle only at P, because every other point of the line is farther from O than the radius. An alternative one-arc method, acceptable when the radius is drawn first, is to construct a right angle at P directly with compasses (the standard angle construction) and produce its side. Figure 3 shows the finished result, with the small square marking the right angle between OP and the tangent — a detail students should always add, since it announces the theorem being used.

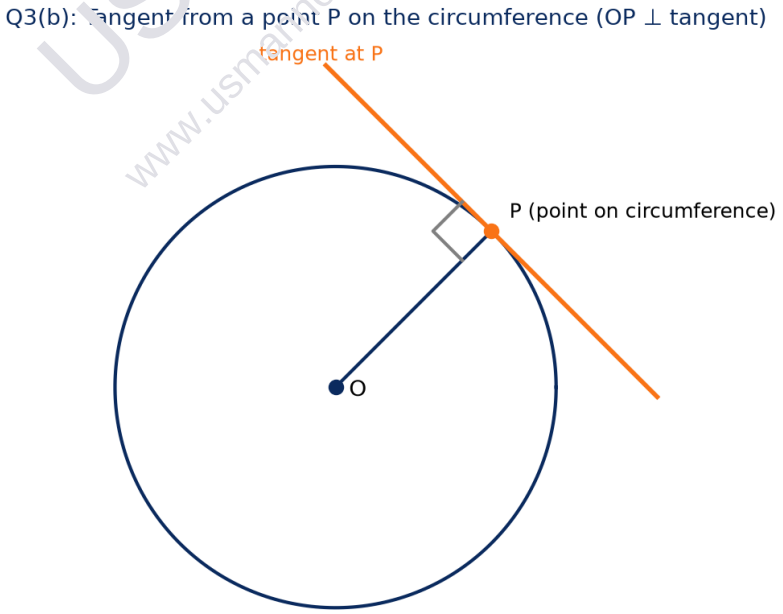


Figure 3: The tangent at a point P on the circumference is the line through P perpendicular to the radius OP.

Variations the Student Should Be Able to Handle

Two variations complete the picture. First, if the two unequal circles touch internally — the smaller inside the larger, touching from within — the construction of the common tangent at the touching point T is identical: T still lies on the line of centres, the tangent is still the perpendicular to that line at T, and the same compass construction erects it. Only the figure changes, not the method, because the underlying theorem (radius to the point of contact perpendicular to the tangent) does not care on which side of T the circles' interiors lie. Second, examiners sometimes ask for a tangent from a point outside the circle; that is a different, longer construction (it uses the circle on the line joining the external point to the centre as diameter) and must not be confused with part (b), where the point is on the circumference and one perpendicular construction suffices. Practical marking points for both parts: use a hard, sharp pencil; draw construction arcs long enough to intersect decisively and do not rub them out; label O_1, O_2, T and P; mark the right angle at the point of contact; and finish with one sentence stating the theorem that justifies the drawing. A neat, labelled figure with visible arcs is what converts a correct idea into full marks in a construction question.

Conclusion

Both constructions rest on the single fact that the radius to the point of contact is perpendicular to the tangent. Two unequal touching circles share a common tangent at their touching point T — the perpendicular to the line of centres at T — and the tangent at a point P on a circumference is the perpendicular to OP at P. Drawn carefully with the construction arcs left in, labelled centres and a marked right angle, these two figures earn the full marks of this question.

Question 4 (20 Marks)

Question 4: (a) The area of a square playground is 4500 m². How long will a man take to cross it diagonally at the speed of 3km per hour? (b) Find the hypotenuse of the right isosceles triangle, each of whose legs is 10cm. (20)

Introduction

Both parts are applications of the theorem of Pythagoras a² + b² = c² for a right-angled triangle. Part (a) hides the triangle inside a square: the diagonal of a square splits it into two right-angled isosceles triangles, so the walking distance is the hypotenuse of a triangle whose legs are the sides of the playground — but the side itself is not given, only the area, so the problem has three stages (area to side, side to diagonal, distance and speed to time) with a unit conversion from metres to kilometres at the end. Part (b) is the pure form of the same theorem for the special case in which the two legs are equal. Keeping the exact surd form until the final step, and only then approximating, is the accurate way to work both parts.

Part (a): Crossing the Square Playground Diagonally

Stage 1 — the side. If the side of the square is s metres, then $s^2 = 4500$, so $s = \sqrt{4500} = \sqrt{(900 \times 5)} = 30\sqrt{5} \approx 67.082$ m. Stage 2 — the diagonal. By Pythagoras, the diagonal d satisfies $d^2 = s^2 + s^2 = 2 \times 4500 = 9000$, so $d = \sqrt{9000} = 30\sqrt{10} \approx 94.868$ m. (Equivalently, diagonal = side × √2 = $30\sqrt{5} \times \sqrt{2} = 30\sqrt{10}$.) Stage 3 — the time. The speed is 3 km per hour = 3000 m per hour, and the distance is 94.868 m = 0.094868 km. Time = distance ÷ speed = 0.094868 ÷ 3 hours ≈ 0.031623 hours. Converting: 0.031623 × 60 ≈ 1.897 minutes, that is about 1 minute 54 seconds (about 114 seconds). So the man takes approximately 1.9 minutes to cross the playground diagonally at 3 km/h. Sanity check: 3 km/h is a gentle walking pace of 50 m per minute, and the diagonal is just under 95 m, so a time of just under two minutes is exactly what common sense expects — a useful confirmation that no factor of 1000 has been lost between metres and kilometres, the

classic error in this standard problem. Students should present the answer in minutes (approximately 1.90 minutes), since the fractional hour 0.0316 h, while correct, conveys little.

Part (b): Hypotenuse of the Right Isosceles Triangle

In a right isosceles triangle the two legs (the sides forming the right angle) are equal; here each is 10 cm. Let the hypotenuse be h. By the theorem of Pythagoras, $h^2 = 10^2 + 10^2 = 100 + 100 = 200$, so $h = \sqrt{200} = 10\sqrt{2} \approx 14.14$ cm. The exact answer $10\sqrt{2}$ cm should be given first, with the decimal 14.14 cm as its approximation. The result illustrates the general pattern worth remembering: in a right isosceles triangle with legs a, the hypotenuse is always $a\sqrt{2}$, and the acute angles are each 45°, so the side ratios are $1 : 1 : \sqrt{2}$. This is the same $\sqrt{2}$ factor that appeared in the diagonal of the square in part (a) — no accident, since the diagonal divides the square into two right isosceles triangles identical in shape to this one. A quick reverse check: $(10\sqrt{2})^2 = 100 \times 2 = 200 = 10^2 + 10^2$, as Pythagoras requires.

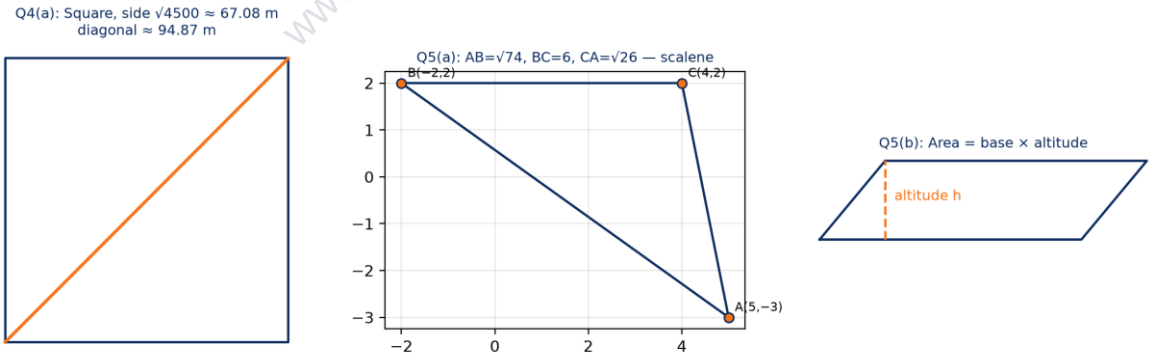


Figure 4: Left — the square playground and its diagonal (Question 4a). Centre — the plotted points of Question 5(a). Right — a parallelogram with its altitude (Question 5b).

Exact Forms, Rounding, and the Special Triangle

Both parts reward keeping exact surds until the end. In part (a), rounding the side to 67 m before finding the diagonal would give d ≈ 94.75 m and a noticeably different time; carrying $30\sqrt{10}$ exactly and approximating once, at the final division, keeps the answer honest at 1 minute 54

seconds to the nearest second. The conversion chain also deserves a clean layout in the student's answer: 3 km/h = 3000 m / 3600 s = 5/6 m per second ≈ 0.833 m/s, and time = 94.868 ÷ 0.833 ≈ 113.8 s gives the same result by the metres-and-seconds route — two routes, one answer. In part (b), the triangle with legs 10, 10 and hypotenuse $10\sqrt{2}$ is the 45°–45°–90° special triangle, whose ratios 1 : 1 : √2 are worth memorising alongside the 30°–60°–90° ratios 1 : √3 : 2, because they convert many examination questions into one-line answers. Notice too that part (a) secretly contains this same triangle: the diagonal of any square makes two 45° angles with the sides, so diagonal = side × √2 is exactly the part (b) result wearing a different costume. Seeing one theorem inside two differently-worded questions is the deepest skill this assignment teaches.

A last practical note on presentation: in part (a) the examiner expects to see the three stages labelled — side, diagonal, time — because each stage carries its own mark, and a single unexplained final number earns only the last of them. Writing the conversion 3 km/h = 50 m per minute explicitly is likewise worth a mark, since unit conversion is part of what the question tests. In part (b), students should state the theorem by name (the theorem of Pythagoras) before substituting, draw a small labelled right-angled triangle with the right angle marked, and box the final results — approximately 1 minute 54 seconds for the crossing, and $10\sqrt{2}$ cm, approximately 14.14 cm, for the hypotenuse — so that the marker can award every step without hunting for it. These presentational habits cost nothing and protect the marks that the mathematics has already earned.

Conclusion

The playground's side is $30\sqrt{5} \approx 67.08$ m and its diagonal is $30\sqrt{10} \approx 94.87$ m, which at 3 km/h takes about 0.0316 hours, i.e. approximately 1 minute 54 seconds, to cross. The hypotenuse of the right isosceles triangle with legs 10 cm is $10\sqrt{2} \approx 14.14$ cm. Both answers follow from

Pythagoras, and both were sanity-checked — the walking time against the familiar 50 m per minute pace, and the hypotenuse by squaring back.

Question 5 (20 Marks)

Question 5: (a) Show that the points A(5, −3), B(−2,2) and C(4,2) These are the vertices of a scalene triangle. (b) The area of a parallelogram is 144 cm². Find the altitude if the base is 2m long. (20)

Introduction

Part (a) belongs to coordinate geometry: three points are given by their coordinates, and the claim is that the triangle they form is scalene, meaning that all three of its sides have different lengths. The distance formula — itself Pythagoras applied to the horizontal and vertical differences of coordinates — turns the claim into three computations, and the proof is finished by comparing the three lengths and checking that the points are not collinear. (The booklet's capital 'These' in mid-sentence is a printing slip, quoted exactly above; it does not affect the mathematics.) Part (b) is a one-formula problem, area of a parallelogram = base × altitude, but it contains a genuine units trap: the area is printed in square centimetres while the base is printed in metres. Handled honestly, with the conversion shown, the printed data give a very small altitude, and this answer presents that result, flags the mismatch openly, and states the answer the question almost certainly intended, clearly labelled.

Part (a): Three Different Side Lengths

The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$. Side AB: A(5, −3) to B(−2, 2): differences −7 and 5, so AB = $\sqrt{(49 + 25)} = \sqrt{74} \approx 8.602$. Side BC: B(−2, 2) to C(4, 2): differences 6 and 0, so BC = $\sqrt{(36 + 0)} = 6$ exactly — a horizontal side, as the equal y-coordinates promise. Side CA: C(4, 2) to A(5, −3): differences 1 and −5, so CA = $\sqrt{(1 + 25)}$ =

√26 ≈ 5.099. The three lengths √74, 6 and √26 are all different: √74 ≠ 6 (since 74 ≠ 36), 6 ≠ √26 (since 36 ≠ 26) and √26 ≠ √74. A triangle with all three sides unequal is by definition a scalene triangle, so A, B and C are the vertices of a scalene triangle, as required. Two supporting checks complete the proof properly. First, the points are not collinear, so they genuinely form a triangle: B and C lie on the horizontal line y = 2 while A has y = −3, so A is off that line. Second, the triangle inequality holds for the lengths: √26 + 6 ≈ 11.10 > √74 ≈ 8.60, and the other two comparisons are equally satisfied. The centre panel of Figure 4 plots the three points so the student can see the unequal sides at a glance.

Part (b): Altitude of the Parallelogram — With a Units Warning

For a parallelogram, Area = base × altitude, so altitude = Area ÷ base, provided both are measured in consistent units. The booklet prints the area as 144 cm² and the base as 2 m. Converting the base to centimetres, 2 m = 200 cm, gives altitude = 144 ÷ 200 = 0.72 cm. That is the mathematically correct answer to the question exactly as printed, and it is the answer this guide records first. However, an altitude of 0.72 cm on a base of 2 m describes an extremely long, extremely flat parallelogram, which is plainly not the picture a Matric textbook intends; the question almost certainly meant the base to be in centimetres as well. Students should therefore add one honest sentence to their answer: 'As printed, with the base converted from metres, the altitude is 0.72 cm; if the base was intended to be 12 cm the altitude would be 12 cm, and if 2 cm was intended the altitude would be 72 cm — the printed units give 0.72 cm.' Note that no 'intended' value can be stated as certain, because several different slips could produce the printed line; what is certain is the method, altitude = area ÷ base with matching units, and the as-printed result 0.72 cm. The general lesson extends well beyond this question: before substituting into any

mensuration formula, convert every quantity to one unit system, and if the answer then looks physically absurd, re-read the question rather than hiding the difficulty.

Side / quantity	Computation	Value
AB	$\sqrt{[(-2-5)^2+(2+3)^2]} = \sqrt{74}$	≈ 8.60
BC	$\sqrt{[(4+2)^2+(2-2)^2]} = \sqrt{36}$	6
CA	$\sqrt{[(5-4)^2+(-3-2)^2]} = \sqrt{26}$	≈ 5.10
Parallelogram altitude (as printed)	144 cm² ÷ 200 cm	0.72 cm

Enrichment: Area of the Triangle, and a Classification Check

The coordinates of part (a) support two further checks that a strong student can add. The area of triangle ABC follows at once because BC is horizontal with length 6 and the perpendicular distance from A(5, −3) to the line y = 2 is $|2-(-3)| = 5$, so area = ½ × base × height = ½ × 6 × 5 = 15 square units; the shoelace formula applied to the three coordinate pairs gives the same 15, confirming the figure. Also, the triangle is scalene but it is worth checking whether it is additionally right-angled: the squares of the sides are 74, 36 and 26, and no two of these sum to the third ($26 + 36 = 62 \neq 74$; $26 + 74$ and $36 + 74$ exceed the remaining side trivially), so by the converse of Pythagoras the triangle is not right-angled at any vertex — its angles are in fact approximately 79°, 36° and 65° by the cosine rule, all different, as befits a scalene triangle. For part (b), the enrichment is a habit rather than a computation: mensuration answers should always end with units, and the unit itself is the first alarm bell — an altitude quoted in centimetres from data mixed between centimetres and metres should always prompt the conversion line '2 m = 200

cm' to be written out in full, exactly as the main answer does, so that a marker can see the trap was noticed and handled deliberately rather than stumbled over.

Conclusion

The distances AB = $\sqrt{74} \approx 8.60$, BC = 6 and CA = $\sqrt{26} \approx 5.10$ are all different, and the points are not collinear, so A(5, −3), B(−2, 2) and C(4, 2) are the vertices of a scalene triangle. For the parallelogram, altitude = area ÷ base gives 0.72 cm from the data exactly as printed (144 cm² and 2 m = 200 cm), with the booklet's centimetre/metre mismatch flagged openly rather than silently corrected. This completes Assignment No. 2, whose recurring lessons match those of Assignment No. 1: name the method requested, show every step, convert units before computing, and verify the final answer against the original statement.

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