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Mathematics-II (248)

Level: Matric

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Assignment No. 1 (Units 1 to 5)

Total Marks: 100 Pass Marks: 40

SOLVED ASSIGNMENT GUIDE — For Study & Reference

Note for Students: This guide is for study and reference only. Read each model answer, understand the method, and then write the assignment in your own words and your own hand. Copying any guide word for word is against the University's plagiarism warning printed in the question booklet, so always rewrite the working in your own style after you have understood it.

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Question 1 (20 Marks)

Question 1: (a) Factorize $x^4 + 64$. (b) Solve $(x + 5)^2 + (2x + 1)^2 - 67 = (x + 5)(2x - 1)$. (20)

Introduction

This question brings together the two most basic skills of Matric algebra: splitting an expression into factors, and solving an equation by first reducing it to standard form. Part (a) looks impossible at first sight, because $x^4 + 64$ is a sum of two squares and a sum of squares does not split over the real numbers in the simple way a difference of squares does. The rescue is an old and beautiful device, sometimes linked with the name of Sophie Germain: add and subtract the same middle term so that the expression becomes a perfect square minus another perfect square,

and then use the difference-of-squares identity. Part (b) is an equation in disguise. Both sides contain brackets, and the safe routine is to expand everything, collect all the terms on one side, and only then solve the resulting quadratic. Students lose marks in such questions not because the ideas are hard, but because they try to cancel or divide before the equation is in standard form. This answer therefore shows every expansion in full, checks each factorisation by multiplying back, and verifies both roots of part (b) by substitution into the original equation.

Part (a): Factorising $x^4 + 64$ by Completing the Square

Write 64 as 8^2, so the expression is $x^4 + 8^2$, a sum of two even powers. The perfect square built on x^2 and 8 would be $(x^2 + 8)^2 = x^4 + 16x^2 + 64$, which contains exactly our two terms plus an extra $16x^2$. So we add and subtract $16x^2$, which changes nothing: $x^4 + 64 = x^4 + 16x^2 + 64 - 16x^2 = (x^2 + 8)^2 - (4x)^2$. Now this is a difference of two squares, $A^2 - B^2$ with $A = x^2 + 8$ and $B = 4x$, and the identity $A^2 - B^2 = (A - B)(A + B)$ gives $x^4 + 64 = (x^2 - 4x + 8)(x^2 + 4x + 8)$. Neither quadratic factor splits further over the real numbers, because each has discriminant $16 - 32 = -16$, which is negative; a negative discriminant means no real linear factors exist, so the factorisation is complete. The check is quick and should always be done: $(x^2 + 8)^2 - (4x)^2$ expands back to $x^4 + 16x^2 + 64 - 16x^2 = x^4 + 64$, exactly the starting expression. A useful way to remember the pattern is that $x^4 + 4b^4$ always factorises in this style; here $64 = 4 \times 16 = 4 \times 2^4$, which is why the trick works so neatly with whole numbers.

Part (b): Expanding and Solving the Equation

Expand each bracket carefully. $(x + 5)^2 = x^2 + 10x + 25$. $(2x + 1)^2 = 4x^2 + 4x + 1$. The right-hand side is $(x + 5)(2x - 1) = 2x^2 - x + 10x - 5 = 2x^2 + 9x - 5$. Putting the expansions into the equation gives $x^2 + 10x + 25 + 4x^2 + 4x + 1 - 67 = 2x^2 + 9x - 5$, that is, $5x^2 + 14x - 41 = 2x^2 + 9x - 5$. Bringing everything to the left gives $3x^2 + 5x - 36 = 0$, a clean quadratic with $a = 3$, $b = 5$

and c = −36. Its discriminant is b² − 4ac = 25 + 432 = 457. Since 457 is not a perfect square (indeed it is a prime number), the quadratic does not factorise with whole numbers, and the honest method is the quadratic formula: x = [−5 ± √457] / 6. With √457 ≈ 21.3776, the two roots are x ≈ (−5 + 21.3776)/6 ≈ 2.7296 and x ≈ (−5 − 21.3776)/6 ≈ −4.3963. Exact answers should be left in surd form, x = (−5 + √457)/6 or x = (−5 − √457)/6, with the decimals given only as approximations.

Verification and Common Errors

Verification by substitution is the student's safety net. Take x ≈ 2.7296 in the original equation: (x + 5)² ≈ (7.7296)² ≈ 59.747, (2x − 1)² ≈ (6.4592)² ≈ 41.721, so the left side is about 59.747 + 41.721 − 67 = 34.468. The right side is (x + 5)(2x − 1) ≈ 7.7296 × 4.4592 ≈ 34.468. The two sides agree, so the root is correct; the negative root checks in exactly the same way. Three errors are common here. First, expanding (2x + 1)² as 4x² + 1 and dropping the middle term 4x. Second, mishandling the minus sign when moving 2x² + 9x − 5 across the equals sign. Third, giving up when the discriminant is not a perfect square: a non-square discriminant does not mean 'no solution'; it means the solutions are irrational and the formula, not factorisation, is the right tool. The summary table below collects the key stages of both parts.

Stage	Part (a)	Part (b)
Key identity / step	(x² + 8)² − (4x)², difference of squares	Expansion, then 3x² + 5x − 36 = 0
Final result	(x² − 4x + 8)(x² + 4x + 8)	x = (−5 ± √457)/6 ≈ 2.73 or −4.40
Check	Multiply back to x⁴ + 64	Substitution makes both sides ≈ 34.47

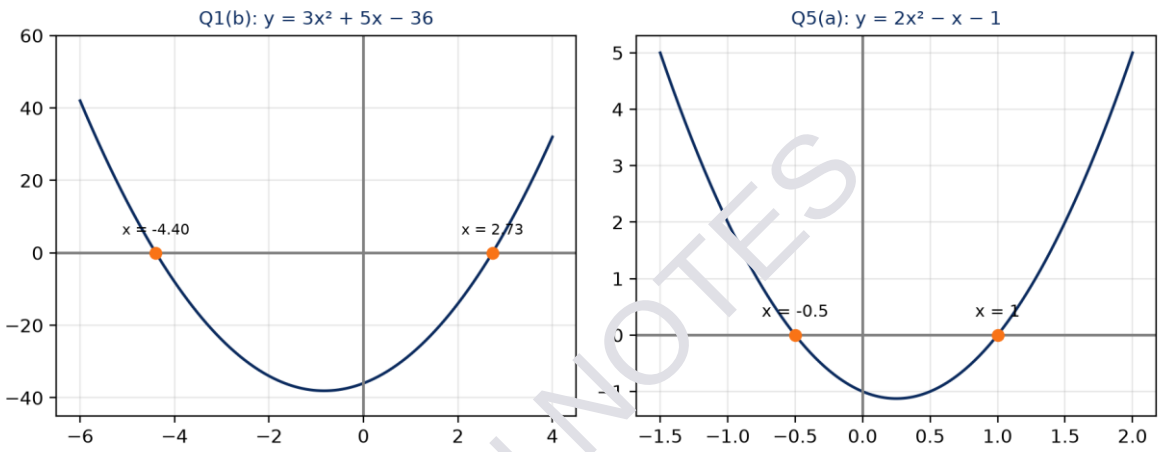


Figure 1: The parabolas of Question 1(b) (left) and Question 5(a) (right); the points where each curve crosses the x-axis are the solutions of the equation.

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A Second Look: The General Pattern Behind Part (a)

The device used in part (a) is worth isolating, because it recurs throughout algebra. The identity $a^4 + 4b^4 = (a^2 - 2ab + 2b^2)(a^2 + 2ab + 2b^2)$ is known as the Sophie Germain identity. Our expression fits it with a = x and b = 2, since $4b^4 = 4 \times 16 = 64$: the factors become $(x^2 - 4x + 8)$ and $(x^2 + 4x + 8)$, exactly as obtained. The same add-and-subtract idea factorises $x^4 + 1$ over the reals in surd form, and it reappears in Question 5(b) of this assignment inside $x^4 + x^2y^2 + y^4$. For part (b), a final discipline point: the two roots $(-5 \pm \sqrt{457})/6$ are irrational, and writing them as decimals too early would corrupt any later use of them — for example, if a later part asked for the sum and product of the roots, the exact values give sum = −5/3 and product = −36/3 = −12 immediately from −b/a and c/a for $3x^2 + 5x - 36 = 0$, whereas rounded decimals give only approximations. Keep surds exact until the final line of a question, and give the decimal only as a check on size: here one root lies between 2 and 3 and the other between −5 and −4, exactly where the graph in Figure 1 crosses the axis.

Conclusion

Part (a) shows that a sum of fourth powers can still be factorised once a completing-the-square term is added and subtracted, giving $(x^2 - 4x + 8)(x^2 + 4x + 8)$. Part (b) shows the standard discipline for bracketed equations: expand fully, reduce to $3x^2 + 5x - 36 = 0$, and apply the quadratic formula to obtain $x = (-5 \pm \sqrt{457})/6$. Both results were verified — the factors by multiplying back and the roots by substitution — which is the habit that protects marks in the examination.

Question 2 (20 Marks)

Question 2: (a) If $x = 2 + \sqrt{5}$, then find the values (i) $x + 1/x$ (ii) $x^2 + 1/x^2$. (b) Find the H.C.F. of $2x^3 + 6x^2 + 5x + 2$, $5x^3 + 10x^2 + 3x - 6$ and $3x^3 + 6x^2 - 2x + 4$. (20)

Introduction

Part (a) is a classic surd problem: a number containing a square root is given, and symmetric expressions in x and its reciprocal must be evaluated exactly. The secret is that although x itself is irrational, the combinations $x + 1/x$ and $x^2 + 1/x^2$ turn out to be beautifully simple, because the surd parts cancel. Part (b) asks for the highest common factor (H.C.F.) of three cubic polynomials, which is found by factorising each one completely and keeping only the factors common to all. Part (b) also contains a printing problem in the booklet, dealt with openly in a transparency note below: worked exactly as printed, the three polynomials share no non-constant factor at all, and this answer shows the working that proves it, then identifies the single sign that has almost certainly been misprinted and gives the intended answer as well, clearly labelled.

Part (a): Working with $x = 2 + \sqrt{5}$

First find $1/x$ by rationalising the denominator. $1/x = 1/(2 + \sqrt{5})$; multiply numerator and denominator by the conjugate $(2 - \sqrt{5})$: $1/x = (2 - \sqrt{5}) / [(2 + \sqrt{5})(2 - \sqrt{5})] = (2 - \sqrt{5}) / (4 - 5) =$

$(2 - \sqrt{5}) / (-1) = \sqrt{5} - 2$. So the reciprocal is simply $\sqrt{5} - 2 \approx 0.23607$, a small positive number, which is sensible because $x \approx 4.23607$ is fairly large. Now (i) $x + 1/x = (2 + \sqrt{5}) + (\sqrt{5} - 2) = 2\sqrt{5}$. The whole-number parts cancel and the surds add, giving the exact value $2\sqrt{5} \approx 4.4721$. Then (ii) uses the identity $x^2 + 1/x^2 = (x + 1/x)^2 - 2$, which comes from expanding $(x + 1/x)^2 = x^2 + 2 + 1/x^2$. Therefore $x^2 + 1/x^2 = (2\sqrt{5})^2 - 2 = 20 - 2 = 18$, a plain whole number. As a direct check: $x^2 = (2 + \sqrt{5})^2 = 4 + 4\sqrt{5} + 5 = 9 + 4\sqrt{5}$, and $1/x^2 = (\sqrt{5} - 2)^2 = 5 - 4\sqrt{5} + 4 = 9 - 4\sqrt{5}$, and their sum is 18 exactly. A bonus observation worth remembering is that $x - 1/x = (2 + \sqrt{5}) - (\sqrt{5} - 2) = 4$, so this x satisfies $x - 1/x = 4$, equivalently $x^2 - 4x - 1 = 0$, which the student can confirm by substituting $x = 2 + \sqrt{5}$.

Part (b): Factorising the Three Cubics

Factorise each polynomial by grouping and by testing simple roots. First polynomial: $2x^3 + 6x^2 + 5x + 2$. Testing $x = -2$ gives $-16 + 24 - 10 + 2 = 0$, so $(x + 2)$ is a factor, and division (or grouping after splitting) gives $2x^3 + 6x^2 + 5x + 2 = (x + 2)(2x^2 + 2x + 1)$; the quadratic has discriminant $4 - 8 = -4$ and goes no further. Second polynomial: $5x^3 + 10x^2 - 3x - 6 = 5x^2(x + 2) - 3(x + 2) = (x + 2)(5x^2 - 3)$, by direct grouping. So the first two polynomials share the factor $(x + 2)$, and their own H.C.F. is $x + 2$. Third polynomial, exactly as printed: $3x^3 + 6x^2 - 2x + 4$. Testing $x = -2$ gives $3(-8) + 6(4) - 2(-2) + 4 = -24 + 24 + 4 + 4 = 8$, which is not zero, so $(x + 2)$ is not a factor of the third polynomial. Its possible rational roots $\pm 1, \pm 2, \pm 4, \pm 1/3, \pm 2/3, \pm 4/3$ all fail on testing, and a greatest-common-divisor computation confirms that it shares no non-constant factor with $(x + 2)(2x^2 + 2x + 1)$ or with $(x + 2)(5x^2 - 3)$. Therefore, for the three polynomials exactly as printed, the only common factor is the constant 1, and the H.C.F. is 1.

Transparency Note (booklet misprint): The result above is mathematically forced by the printed text, but it makes a poor examination question, and the cause is easy to locate. The third

polynomial fails to vanish at x = −2 by exactly 8. If the printed '− 2x + 4' were '+ 2x + 4' (one sign changed), the polynomial would be $3x^3 + 6x^2 + 2x + 4 = 3x^2(x + 2) + 2(x + 2) = (x + 2)(3x^2 + 2)$, and the H.C.F. of all three would be the neat answer x + 2 that the question is plainly designed to produce. Students copying this guide should present the factorisations in full, state the H.C.F. as printed (1), and add one line noting the apparent sign misprint and the intended answer x + 2. That is the honest way to handle a defective question: never silently change the question, and never invent a common factor that is not there.

Polynomial	Factorisation	Value at x = −2
$2x^3 + 6x^2 + 5x + 2$	$(x + 2)(2x^2 + 2x + 1)$	0
$5x^3 + 10x^2 - 3x - 6$	$(x + 2)(5x^2 - 3)$	0
$3x^3 + 6x^2 - 2x + 4$ (as printed)	No factor (x + 2); irreducible over the rationals as a cubic with no rational root	8 (not 0)

Extending Part (a) and the Method of Part (b)

The two values found in part (a) generate the whole family of symmetric values. For instance, $x^3 + 1/x^3 = (x + 1/x)^3 - 3(x + 1/x) = (2\sqrt{5})^3 - 3(2\sqrt{5}) = 40\sqrt{5} - 6\sqrt{5} = 34\sqrt{5}$, and $x - 1/x = 4$ was noted earlier; between them, every expression $x^n + 1/x^n$ can be built up step by step, which is why examiners love this question — one small computation unlocks many results. For part (b), the systematic alternative to factorising each polynomial separately is the Euclidean algorithm for polynomials: divide the larger by the smaller, keep the remainder, and repeat until the remainder is zero or constant; the last non-zero remainder is the H.C.F. Applied to the first two polynomials it returns (x + 2), in agreement with the factorisation method. Applied with the third

printed polynomial it terminates in a constant, confirming once more that the H.C.F. of all three, as printed, is 1. Presenting a factorisation, as the main answer does, is preferred at Matric level because it shows the factors explicitly — but knowing that a mechanical check exists is what allows a student to state the awkward answer 1 with confidence instead of assuming a personal error and forcing a neater-looking but false result.

Conclusion

For $x = 2 + \sqrt{5}$, rationalising gives $1/x = \sqrt{5} - 2$, hence $x + 1/x = 2\sqrt{5}$ and $x^2 + 1/x^2 = 18$, both verified by direct expansion. For the H.C.F., the first two polynomials factorise with the common factor $(x + 2)$, but the third polynomial as printed does not contain that factor — it equals 8, not 0, at $x = -2$ — so the H.C.F. of the three printed polynomials is 1, with $x + 2$ recorded as the evidently intended answer if the third polynomial's middle sign is corrected.

Question 3 (20 Marks)

Question 3: (a) Derive the quadratic formula for the quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$. (b) Find two consecutive positive odd numbers given that the difference between their reciprocals is 2/63. (20)

Introduction

Part (a) asks where the quadratic formula comes from. Many students memorise $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$ without knowing that it is nothing more than completing the square carried out once, in letters, for every quadratic at the same time. Deriving it properly is therefore one of the most valuable exercises in the Matric course: it explains the formula, it explains the discriminant, and it explains why $a \neq 0$ must be assumed. Part (b) is a word problem that turns into a quadratic equation: two consecutive odd numbers are described through their reciprocals, and translating

that sentence into algebra, solving, and then rejecting the root that the words 'positive' exclude is the complete skill being tested.

Part (a): Derivation by Completing the Square

Start with $ax^2 + bx + c = 0$ with $a \neq 0$. Step 1: divide every term by a (allowed because $a \neq 0$), giving $x^2 + (b/a)x + c/a = 0$. Step 2: move the constant term to the right: $x^2 + (b/a)x = -c/a$. Step 3: complete the square on the left. Half the coefficient of x is b/2a; add its square, $(b/2a)^2 = b^2/4a^2$, to both sides: $x^2 + (b/a)x + b^2/4a^2 = b^2/4a^2 - c/a$. Step 4: the left side is now a perfect square, $(x + b/2a)^2$, and the right side combines over the common denominator $4a^2$ to give $(b^2 - 4ac)/4a^2$. So $(x + b/2a)^2 = (b^2 - 4ac)/4a^2$. Step 5: take the square root of both sides, remembering that a square root always carries both signs: $x + b/2a = \pm\sqrt{(b^2 - 4ac)}/2a$. Step 6: subtract b/2a from both sides to reach the formula: $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$, which is what was to be derived. The quantity $D = b^2 - 4ac$ under the root is the discriminant: if $D > 0$ there are two distinct real roots, if $D = 0$ the two roots coincide, and if $D < 0$ there are no real roots, because the square root of a negative number is not a real number. Notice how each condition in the question is used: the derivation divides by a twice, which is exactly why the booklet states $a \neq 0$ — if a were 0 the equation would not be quadratic at all, only linear.

Part (b): Two Consecutive Positive Odd Numbers

Let the smaller odd number be n; the next consecutive odd number is then n + 2, because odd numbers are spaced two apart. The difference between their reciprocals is 2/63, and since the smaller number has the larger reciprocal, the equation is $1/n - 1/(n + 2) = 2/63$. The left side combines to $[(n + 2) - n] / [n(n + 2)] = 2 / [n(n + 2)]$. So $2/[n(n + 2)] = 2/63$, and cancelling the 2s and cross-multiplying gives $n(n + 2) = 63$, that is, $n^2 + 2n - 63 = 0$. Factorising, we need two numbers with product −63 and sum +2: these are +9 and −7, so $(n + 9)(n - 7) = 0$ and $n = 7$ or n

= −9. The question demands positive numbers, so n = −9 is rejected (it would give the pair −9 and −7, whose reciprocals do in fact differ by 2/63 as well, but they are negative). The required numbers are therefore 7 and 9. Verification: 1/7 − 1/9 = (9 − 7)/63 = 2/63, exactly as stated. The rejected root is instructive: algebra alone gives both pairs, and only reading the word 'positive' in the question selects the right one — a reminder that in word problems the final sentence of the answer must always be checked against every condition in the wording.

Discriminant $D = b^2 - 4ac$	Nature of roots	Example
$D > 0$	Two distinct real roots	$3x^2 + 5x - 36 = 0$ (Question 1): $D = 457$
$D = 0$	Two equal (repeated) real roots	$x^2 - 6x + 9 = 0$ gives $x = 3, 3$
$D < 0$	No real roots	$x^2 + x + 1 = 0$: $D = -3$

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The Derivation Viewed Another Way, and Choosing a Method

There is a classical variant of the derivation, often called Sridhara's method, which avoids fractions until the last step. Multiply $ax^2 + bx + c = 0$ through by 4a: $4a^2x^2 + 4abx + 4ac = 0$. The first two terms are the start of the square $(2ax + b)^2 = 4a^2x^2 + 4abx + b^2$, so rewriting gives $(2ax + b)^2 = b^2 - 4ac$, whence $2ax + b = \pm\sqrt{(b^2 - 4ac)}$ and $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$ — the same formula, reached with the denominator carried by the 2a from the beginning. Comparing the three solution methods now available to the student: factorisation is fastest when the roots are rational and the factors can be spotted (as in Question 5(a) of this assignment, where the question explicitly demands it); completing the square is the method that also reveals the vertex of the parabola and handles every case with exact working; and the formula is the universal machine,

best reserved for non-factorisable cases such as Question 1(b), where the discriminant 457 was not a perfect square. A worked confirmation of the formula on part (b)'s own equation was already given in the verification there; conversely, the formula applied to 7 and 9's equation $n^2 + 2n - 63 = 0$ gives $n = [-2 \pm \sqrt{(4 + 252)}]/2 = [-2 \pm 16]/2 = 7$ or -9, agreeing with the factorisation used above. Consistency between methods is the hallmark of a correct answer.

Conclusion

Completing the square on $ax^2 + bx + c = 0$ yields the quadratic formula $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$, with the discriminant $b^2 - 4ac$ controlling the nature of the roots. Applied thinking of the same kind solves part (b): the reciprocal condition becomes $n^2 + 2n - 63 = 0$, whose admissible root $n = 7$ gives the consecutive positive odd numbers 7 and 9, verified by $1/7 - 1/9 = 2/63$.

Question 4 (20 Marks)

Question 4: (a) Simplify $(2x^2 - 5x - 12)/(4x^2 + 4x - 3) \times (2x^2 - 7x - 4)/(6x^2 + 5x - 4)$. (b) One algebraic expression is $x^3 - x^2 + 2x - 2$ and the other one is $x^3 - x^2 - 2x + 2$. Their H.C.F. is $x - 1$. Find their L.C.M. (20)

Introduction

Part (a) is a multiplication of two algebraic fractions. The golden rule is the opposite of what beginners do: do not multiply the quadratics out into quartics. Instead, factorise every numerator and denominator first, cancel any factor that appears in both a numerator and a denominator, and multiply only what survives. Part (b) uses the fundamental relation between two expressions and their H.C.F. and L.C.M.: for any two expressions, (first expression) × (second expression) = H.C.F. × L.C.M. Because the H.C.F. is given, the L.C.M. follows by one division — but the wise student also factorises both cubics completely, which both checks the given H.C.F. and produces the L.C.M. directly. Both routes are shown here and, as they must, they agree.

Part (a): Factorise First, Then Cancel

Factorise all four quadratics. $2x^2 - 5x - 12 = (2x + 3)(x - 4)$, since $2x \times x$ gives $2x^2$, the cross terms $3x - 8x$ give $-5x$, and $3 \times (-4) = -12$. $4x^2 + 4x - 3 = (2x + 3)(2x - 1)$, since the cross terms $6x - 2x$ give $+4x$ and $3 \times (-1) = -3$. $2x^2 - 7x - 4 = (2x + 1)(x - 4)$, since the cross terms $x - 8x$ give $-7x$. $6x^2 + 5x - 4 = (2x - 1)(3x + 4)$, since the cross terms $8x - 3x$ give $+5x$. (Each factorisation can be confirmed in seconds by the ac-method: split the middle term using two numbers whose product is $a \times c$ and whose sum is b.) The product is therefore $[(2x + 3)(x - 4) / (2x + 3)(2x - 1)] \times [(2x + 1)(x - 4) / (2x - 1)(3x + 4)]$. The factor $(2x + 3)$ appears in a numerator and a denominator and cancels (for $x \neq -3/2$). Collecting what remains gives the simplified answer $(x - 4)^2(2x + 1) / [(2x - 1)^2(3x + 4)]$, valid wherever the original denominators are non-zero, i.e. $x \neq 1/2$, $x \neq -3/2$ and $x \neq -4/3$. A numerical spot-check protects against slips: at $x = 0$ the original product is $(-12/-3) \times (-4/-4) = 4 \times 1 = 4$, and the simplified form gives $(16 \times 1)/(1 \times 4) = 4$. Agreement at a test value does not prove an identity, but disagreement would disprove it, so the check is always worth the half-minute it takes.

Part (b): L.C.M. from H.C.F. × L.C.M. = Product

Factorise the two cubics by grouping. First expression: $x^3 - x^2 + 2x - 2 = x^2(x - 1) + 2(x - 1) = (x - 1)(x^2 + 2)$. Second expression: $x^3 - x^2 - 2x + 2 = x^2(x - 1) - 2(x - 1) = (x - 1)(x^2 - 2)$. The common factor is indeed $(x - 1)$, confirming the H.C.F. stated in the question. The L.C.M. must contain every factor that appears in either expression, taken at its highest power: $(x - 1)$ appears in both, $(x^2 + 2)$ only in the first and $(x^2 - 2)$ only in the second, so L.C.M. $= (x - 1)(x^2 + 2)(x^2 - 2) = (x - 1)(x^4 - 4) = x^5 - x^4 - 4x + 4$. The formula route gives the same thing in one line: L.C.M. = (first × second) / H.C.F. $= [(x - 1)(x^2 + 2)(x - 1)(x^2 - 2)] / (x - 1) = (x - 1)(x^2 + 2)(x^2 - 2)$. Note that $x^2 - 2$ could be split further as $(x - \sqrt{2})(x + \sqrt{2})$ over the reals, but in Matric

polynomial work factors are kept rational, so $(x^2 - 2)$ and the irreducible $(x^2 + 2)$ are left as they are. The degree check also makes sense: the L.C.M. of two cubics sharing one linear factor has degree $3 + 3 - 1 = 5$, matching $x^5 - x^4 - 4x + 4$.

Expression	Factorised form
$2x^2 - 5x - 12$	$(2x + 3)(x - 4)$
$4x^2 + 4x - 3$	$(2x + 3)(2x - 1)$
$2x^2 - 7x - 4$	$(2x + 1)(x - 4)$
$6x^2 + 5x - 4$	$(2x - 1)(3x + 4)$
$x^3 - x^2 + 2x - 2$	$(x - 1)(x^2 + 2)$
$x^3 - x^2 - 2x + 2$	$(x - 1)(x^2 - 2)$

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Restrictions, and Why the H.C.F.–L.C.M. Rule Works

Two refinements complete part (a). First, cancelling a factor is legitimate only where that factor is not zero, which is why the exclusions $x \neq 1/2, -3/2, -4/3$ were recorded: at those values an original denominator vanishes and the original expression simply does not exist, even though the simplified form might give a number for some of them. Examination answers that state the simplified fraction together with its restrictions are the complete ones. Second, a further test value guards the algebra: at $x = 1$ the original product is $[(2 - 5 - 12)/(4 + 4 - 3)] \times [(2 - 7 - 4)/(6 + 5 - 4)] = (-15/5) \times (-9/7) = 27/7 \approx 3.857$, and the simplified form gives $[(1 - 4)^2(3)] / [(1)^2(7)] = 27/7$ — agreement at a second, unrelated point. For part (b), the rule H.C.F. × L.C.M. = product holds for two expressions for exactly the reason it holds for two whole numbers: every prime (here, irreducible) factor appears in the product with the sum of its two exponents, in the H.C.F. with the smaller exponent and in the L.C.M. with the larger, and smaller + larger = sum.

It is a two-expression rule; for three expressions it fails in general, which is why Question 2(b) of this assignment had to proceed by common factors directly. Finally, note the shape of the answer: $(x^2 + 2)$ can never vanish for real x, while $(x^2 - 2)$ vanishes at $x = \pm\sqrt{2}$; neither is split at this level, because polynomial H.C.F. and L.C.M. work is done over the rational numbers unless a question says otherwise.

Conclusion

Factorising first reduces the fraction product to $(x - 4)^2(2x + 1) / [(2x - 1)^2(3x + 4)]$ after cancelling $(2x + 3)$, and a test value confirms it. For the two cubics, complete factorisation confirms the given H.C.F. $(x - 1)$ and yields L.C.M. $= (x - 1)(x^2 + 2)(x^2 - 2) = x^5 - x^4 - 4x + 4$, in full agreement with the product rule H.C.F. × L.C.M. = product of the two expressions.

Question 5 (20 Marks)

Question 5: (a) Solve $4x(3x - 1) - 2 = (2x - 1)(5x + 1)$ by using the factorization method. (b) Find the continued product of $x^6 - y^6$. (20)

Introduction

Part (a) specifically orders the factorisation method, so the quadratic formula, although it would give the same roots, is not what is being examined: the marker wants to see the equation reduced to zero on one side, split into two linear factors, and finished with the zero-product rule. Part (b) uses the phrase 'continued product', which in the Matric textbook means factorising completely, step by step, until every factor can go no further — for $x^6 - y^6$ this is a showcase of the difference of squares, the difference of cubes and the sum of cubes identities used in sequence. Together the two parts test whether the student commands the standard identities fluently rather than mechanically.

Part (a): Solving by Factorisation

Expand both sides. Left side: $4x(3x - 1) - 2 = 12x^2 - 4x - 2$. Right side: $(2x - 1)(5x + 1) = 10x^2 + 2x - 5x - 1 = 10x^2 - 3x - 1$. Move everything to the left: $12x^2 - 4x - 2 - 10x^2 + 3x + 1 = 0$, giving $2x^2 - x - 1 = 0$. Now factorise: we need to split the middle term $-x$ into two parts whose coefficients have product $2 \times (-1) = -2$ and sum -1; the pair -2 and $+1$ does this. So $2x^2 - 2x + x - 1 = 2x(x - 1) + 1(x - 1) = (2x + 1)(x - 1) = 0$. By the zero-product rule, either $2x + 1 = 0$, giving $x = -1/2$, or $x - 1 = 0$, giving $x = 1$. Verification: for $x = 1$, the left side is $4(1)(2) - 2 = 6$ and the right side is $(1)(6) = 6$. For $x = -1/2$, the left side is $4(-1/2)(-5/2) - 2 = 5 - 2 = 3$ and the right side is $(-2)(-3/2) = 3$. Both roots satisfy the original equation, so the solution set is $x = 1$ or $x = -1/2$. The graph in Figure 1 (right panel) shows the same story geometrically: the parabola $y = 2x^2 - x - 1$ crosses the x-axis exactly at $-1/2$ and 1.

Part (b): The Continued Product of $x^6 - y^6$

Treat $x^6 - y^6$ first as a difference of squares, $(x^3)^2 - (y^3)^2$: $x^6 - y^6 = (x^3 - y^3)(x^3 + y^3)$. Now apply the cube identities. The difference of cubes gives $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$, and the sum of cubes gives $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$. Combining, the continued product is $x^6 - y^6 = (x - y)(x + y)(x^2 + xy + y^2)(x^2 - xy + y^2)$. The two quadratic factors cannot be split further over the real numbers (each has a negative discriminant when viewed as a quadratic), so the factorisation is complete. There is an equally valid alternative order: start by writing $x^6 - y^6 = (x^2)^3 - (y^2)^3$ as a difference of cubes, giving $(x^2 - y^2)(x^4 + x^2y^2 + y^4)$, then split $x^2 - y^2 = (x - y)(x + y)$ and the quartic as $x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2)$ by the same add-and-subtract device used in Question 1(a) — the final product is identical, as it must be, because complete factorisation into irreducibles is unique. A quick check with numbers, $x = 2$ and $y = 1$: $x^6 - y^6 = 64 - 1 = 63$, and the product gives $(1)(3)(4 + 2 + 1)(4 - 2 + 1) = 3 \times 7 \times 3 = 63$. Correct.

Identity	Form used in this question

Difference of squares	$A^2 - B^2 = (A - B)(A + B)$, with $A = x^3$, $B = y^3$
Difference of cubes	$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
Sum of cubes	$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
Zero-product rule (part a)	If $(2x + 1)(x - 1) = 0$ then $2x + 1 = 0$ or $x - 1 = 0$

What 'Continued Product' Means, and Final Checks

The phrase 'continued product' deserves a precise explanation, because students sometimes stop halfway. It means: continue factorising, applying identity after identity, until every factor is irreducible — no factor can be split further over the rational numbers (and, for the two quadratics here, over the real numbers either, as their discriminants are negative). Stopping at $(x^3 - y^3)(x^3 + y^3)$ earns only part of the marks; stopping at $(x - y)(x + y)(x^2 + xy + y^2)(x^2 - xy + y^2)$ completes it. A symmetry check confirms the result at a glance: the left side $x^6 - y^6$ changes sign if x and y are swapped, and the product does too, because exactly one factor, $(x - y)$, changes sign while $(x + y)$, and the two quadratics, are symmetric. A second numerical check at $x = 3$, $y = 2$ gives $729 - 64 = 665$ on the left and $(1)(5)(9 + 6 + 4)(9 - 6 + 4) = 5 \times 19 \times 7 = 665$ on the right. For part (a), the corresponding final discipline is to resist decimal answers entirely: the equation has rational roots, the factorisation method finds them exactly, and any answer written as decimals would reveal that the prescribed method had not really been used. Between them, the two parts reward the same habit — carry the standard identities accurately, in the right order, all the way to irreducible factors or exact roots.

Conclusion

By the prescribed factorisation method, $4x(3x - 1) - 2 = (2x - 1)(5x + 1)$ reduces to $(2x + 1)(x - 1) = 0$, so $x = 1$ or $x = -1/2$, both verified in the original equation. The continued product of $x^6 - y^6$, obtained by the difference of squares followed by the sum and difference of cubes, is $(x - y)(x + y)(x^2 + xy + y^2)(x^2 - xy + y^2)$, confirmed numerically. This closes Assignment No. 1: across its five questions the recurring lessons are to expand and factorise carefully, to verify every result, and to read any apparently defective printed question honestly rather than forcing an answer.

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