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Semester: Autumn 2026

ALLAMA IQBAL OPEN UNIVERSITY

Department of Mathematics

Course: Mathematics-I (247)

Programme / Level: Matric

Semester: Autumn 2026

Assignment No. 2 (Units 6 to 9)

Total Marks: 100 | Pass Marks: 40

SOLVED ASSIGNMENT GUIDE — For Study & Reference

Note: This guide is for study and reference only. Students are advised to understand the methods shown and to write their assignments in their own words; copying verbatim is discouraged and the university's plagiarism policy applies to submitted work.

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Notes on the Printed Booklet

This booklet (Course: Mathematics-I (247), Level: Matric, Semester: Autumn, 2026, Department of Mathematics) contains Assignments No. 1 and No. 2 only, each of 100 total marks with 40 pass marks and five questions of 20 marks each; Assignment No. 2 covers Units 6 to 9. No question in Assignment No. 2 is misprinted: the relation in Question 1(b) is printed with the condition y <= x, the fractions and exponents in Question 2 and Question 4 were read from the printed page, and Question 4 is answered with drawn graphs of both lines, embedded as figures, together with the intercept working that fixes each line. The illustrative numerical examples used to display the general formulas in Questions 1(a) and 3(b) are labelled as illustrative inside the answers; they are not part of the booklet's questions. All numerical answers in this guide were

worked and cross-checked by computer, and the logarithmic evaluation in Question 2(a) was verified against a direct computation (1,963.79), the table method giving approximately 1,964.

Question 1

Question 1 (20 marks): (a) Insert n G.Ms between two numbers a and b. (b) If L = {1,2,3} and M = {2,3,4} Then write a binary relation R such that R = {(x, y)| x in L, y in M and y <= x}. Also write Dom (R) and Range (R).

Introduction

This question joins a standard derivation from the theory of sequences, the insertion of geometric means between two given numbers, with an elementary exercise on sets and relations, the construction of a binary relation from a stated condition. The two parts share a common discipline: each begins from a precise definition and lets the required objects be forced by that definition, rather than guessed. In part (a), to insert n geometric means between a and b means to find n numbers which, placed between a and b, make the whole list of n + 2 numbers a geometric progression; the unknown common ratio is then determined by counting how many steps of multiplication carry us from a to b, and each mean is written down in terms of a, b and its position. In part (b), a binary relation from L to M is any set of ordered pairs whose first element comes from L and whose second from M; here the condition y <= x selects exactly which of the nine possible pairs qualify, and the domain and range are then read off as the sets of first and second elements that actually occur. Both parts are developed from their definitions below, and each is illustrated by a small numerical example, clearly labelled as illustrative, so that the general formulas can be seen working on concrete numbers.

Part (a): Inserting n Geometric Means Between a and b

Let the n geometric means to be inserted be G1, G2, ..., Gn. Then the sequence a, G1, G2, ..., Gn, b is a geometric progression containing n + 2 terms in all, with first term a and last term b. Let r be the common ratio of this progression. The terms are a, ar, ar^2, ..., and in general the k-th term is a r^(k-1). The last term, b, is the (n + 2)-th term, so b = a r^((n+2)-1) = a r^(n+1). Dividing by a (the derivation assumes a is not zero, as a geometric progression in this course is built from non-zero terms), r^(n+1) = b / a, and taking the (n + 1)-th root, r = (b / a)^(1/(n+1)). This single formula is the heart of the answer: inserting n means creates n + 1 equal multiplicative steps between a and b, so the ratio is the (n + 1)-th root of the overall ratio b/a, not, as beginners often write, the n-th root. Once r is known, every mean follows at once: G1 = a r = a (b/a)^(1/(n+1)), G2 = a r^2 = a (b/a)^(2/(n+1)), and in general the k-th geometric mean, for k = 1, 2, ..., n, is Gk = a r^k = a (b/a)^(k/(n+1)), which can also be written in the symmetric form Gk = a^((n+1-k)/(n+1)) b^(k/(n+1)), showing how each mean leans more heavily on b as k grows.

An illustrative example, using numbers chosen by this guide purely to display the method and not taken from the booklet, makes the derivation tangible. Insert 3 geometric means between 2 and 162. Here n = 3, so there are n + 1 = 4 steps and r = (162/2)^(1/4) = 81^(1/4) = 3. The means are 2 x 3 = 6, 2 x 3^2 = 18 and 2 x 3^3 = 54, and indeed 2, 6, 18, 54, 162 is a geometric progression with common ratio 3, ending exactly at 162, which checks the formula. Two further remarks complete the theory at Matric level. First, when n = 1 the single geometric mean between a and b is G1 = a (b/a)^(1/2), the familiar square-root expression whose value squared equals ab, which is why the geometric mean of two positive numbers is the square root of their product. Secondly, if b/a is such that the (n + 1)-th root is not a tidy number, the means are simply left in the surd or fractional-exponent form given by the formula; the answer is still

complete, because the question asks for the means in terms of a and b. Where a and b are both positive, as in the usual textbook setting, the positive real root is taken throughout.

Part (b): Building the Relation R from L to M

The Cartesian product L x M consists of all nine ordered pairs that can be formed with a first element from L = {1, 2, 3} and a second element from M = {2, 3, 4}: (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4). The relation R keeps only those pairs that satisfy the stated condition y <= x, where, by the definition as printed, x is the element of L and y the element of M. Testing each pair in turn: for x = 1, no element of M is less than or equal to 1, so no pair with first element 1 qualifies. For x = 2, the elements of M satisfying y <= 2 form just {2}, giving the pair (2, 2). For x = 3, the elements of M satisfying y <= 3 are 2 and 3, giving the pairs (3, 2) and (3, 3). The element 4 of M never appears, because no element of L is greater than or equal to 4. Hence R = {(2, 2), (3, 2), (3, 3)}. The domain of R, Dom (R), is the set of first elements actually occurring, {2, 3}, and the range of R, Range (R), is the set of second elements actually occurring, {2, 3}. The grid figure below displays all nine candidate pairs and highlights the three selected ones, making the selection rule visible at a glance.

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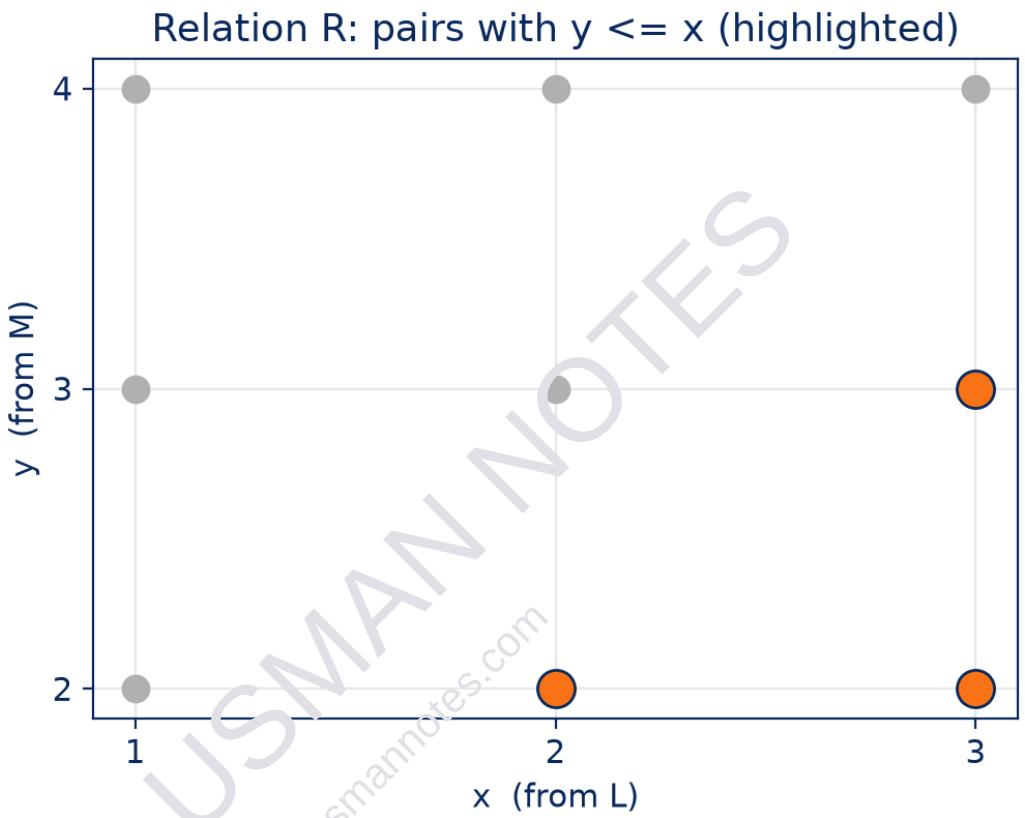


Figure 1: The nine pairs of L x M, with the three pairs satisfying y <= x highlighted.

x (from L)	Values of y in M with y <= x	Pairs admitted to R
1	None	None
2	2	(2, 2)
3	2, 3	(3, 2), (3, 3)

Several teaching points are packed into this short part. A relation is not required to use every element of either set: here 1 belongs to L yet has no partner, and 4 belongs to M yet is never reached, which is precisely why Dom (R) is a subset of L and Range (R) is a subset of M rather than being automatically equal to them. Nor is R a function, because the element 3 of the domain

is paired with two different elements, 2 and 3, of the range; a function would demand exactly one image for each domain element. The order inside each pair matters throughout: (3, 2) belongs to R because 2 <= 3, whereas the reversed pair (2, 3) does not, because 3 is not less than or equal to 2, and students should test the condition exactly as printed, with the first coordinate in the role of x, instead of testing whichever order happens to make an inequality true. Set-builder notation, roster form, domain and range are four descriptions of the same object here, and the answer gives all of them so that the marker can see the complete chain from definition to result.

Conclusion

For part (a), inserting n geometric means between a and b makes a progression of n + 2 terms whose common ratio is r = (b/a)^(1/(n+1)), and the means are Gk = a (b/a)^(k/(n+1)) for k = 1, 2, ..., n, a formula verified on the illustrative case of 6, 18 and 54 between 2 and 162. For part (b), applying the printed condition y <= x to the nine pairs of L x M leaves exactly three pairs, R = {(2, 2), (3, 2), (3, 3)}, with Dom (R) = {2, 3} and Range (R) = {2, 3}. In both parts the answer was obtained not by pattern-spotting but by returning to the definition, of a geometric progression in the first case and of a relation, its domain and its range in the second, and letting the definition dictate every step, which is the most reliable method in the whole of this unit.

Question 2

Question 2 (20 marks): (a) Using a logarithmic table, evaluate [(6.45)^3 x (0.00034)^(1/3) x (981.9)] / [(9.37)^2 x (8.93)^(1/4) x (0.0617)] (b) Prove that 7 log (16/15) + 5 log (25/24) + 3 log (81/80) = log 2.

Introduction

Logarithms turn multiplication into addition, division into subtraction, powers into multiplications and roots into divisions, and this question exercises both directions of that idea. Part (a) asks for the numerical value of a complicated product, quotient, power and root expression, to be found by the classical logarithmic-table method that the Matric course still teaches: take the common logarithm of the whole expression, use the laws of logarithms to break it into a sum of table look-ups, evaluate that sum carefully, handling numbers smaller than 1 by the bar notation for negative characteristics, and finally take the antilogarithm. Part (b) runs the machine in reverse: starting from a sum of logarithms it must be compressed, by the same laws, into the logarithm of a single number, and that number must turn out to be exactly 2. The two parts are therefore mirror images, and the same three laws govern both: log (xy) = log x + log y, log (x / y) = log x - log y, and log (x^n) = n log x. Every logarithm in this answer is to base 10, as the booklet's use of log tables implies, and part (a) is also computed directly by machine in this guide as a check, so that the table result can be compared with the true value.

Part (a): Taking the Logarithm of the Whole Expression

Let x denote the given expression. Taking logarithms, the products in the numerator become sums, the division by the denominator becomes a subtraction, the cube becomes a multiplication by 3, the cube root becomes a division by 3, the square a multiplication by 2, and the fourth root a division by 4. Thus log x = [3 log 6.45 + (1/3) log 0.00034 + log 981.9] - [2 log 9.37 + (1/4) log 8.93 + log 0.0617]. From a four-figure table: log 6.45 = 0.8096, log 981.9 = 2.9921, log 9.37 = 0.9717 and log 8.93 = 0.9509. For the two numbers smaller than 1 the characteristic is negative: 0.00034 = 3.4 x 10^(-4), so log 0.00034 = -4 + log 3.4 = -4 + 0.5315, written in bar notation as bar-4.5315, whose decimal value is -3.4685; and 0.0617 = 6.17 x 10^(-2), so log 0.0617 = bar-2.7903, whose decimal value is -1.2097. Substituting, the numerator contributes

3(0.8096) + (1/3)(-3.4685) + 2.9921 = 2.4288 - 1.1562 + 2.9921 = 4.2647, and the denominator contributes 2(0.9717) + (1/4)(0.9509) + (-1.2097) = 1.9434 + 0.2377 - 1.2097 = 0.9714. Therefore log x = 4.2647 - 0.9714 = 3.2933, to table accuracy.

Quantity	Logarithm (4 figures)	Multiplier	Contribution
6.45	0.8096	x 3	2.4288
0.00034	bar-4.5315 (= -3.4685)	x 1/3	-1.1562
981.9	2.9921	x 1	2.9921
9.37	0.9717	x 2 (subtract)	1.9434
8.93	0.9509	x 1/4 (subtract)	0.2377
0.0617	bar-2.7903 (= -1.2097)	x 1 (subtract)	-1.2097

It remains to undo the logarithm. Since log x = 3.2933, the characteristic 3 shows that x lies between 1,000 and 10,000, and the antilogarithm of the mantissa 0.2933 is approximately 1.964, so x is approximately 1.964 x 10^3, that is about 1,964. A direct machine computation of the original expression gives 1,963.79, so the four-figure table answer of about 1,964 agrees with the true value to within the accuracy that printed tables can deliver; tiny differences in the third or fourth figure between one student's table work and another's are normal and are not errors, because each table entry is itself rounded. The most delicate step of the whole evaluation is the

division of the negative logarithm by 3: bar-4.5315 divided by 3 is most safely handled in ordinary decimal form, -3.4685 / 3 = -1.1562, as done above; students who divide the characteristic and mantissa separately without first making the characteristic exactly divisible by 3, by borrowing from the mantissa in the standard way, are the ones whose answers drift. A rough order-of-magnitude estimate also guards the result: the numerator is roughly 268 x 0.07 x 982, about 18,400, and the denominator roughly 88 x 1.73 x 0.0617, about 9.4, giving a quotient near 1,960, comfortably close to 1,964 and far from any misplacement of the decimal point.

Part (b): Compressing the Sum into log 2

For part (b), apply the power law first, turning each coefficient into an exponent: 7 log (16/15) = log (16/15)^7, 5 log (25/24) = log (25/24)^5, and 3 log (81/80) = log (81/80)^3. The product law then combines the sum into the logarithm of a single product: the left-hand side equals log [(16/15)^7 x (25/24)^5 x (81/80)^3]. Now write every number in prime factors: 16 = 2^4, 15 = 3 x 5, 25 = 5^2, 24 = 2^3 x 3, 81 = 3^4 and 80 = 2^4 x 5. Collecting the exponent of each prime in the whole product: for the prime 2, the contributions are 4 x 7 from the first factor, minus 3 x 5 from the second denominator, minus 4 x 3 from the third denominator, giving 28 - 15 - 12 = 1. For the prime 3, the contributions are minus 7 from the first denominator, minus 5 from the second denominator, plus 4 x 3 = 12 from the third numerator, giving -7 - 5 + 12 = 0. For the prime 5, the contributions are minus 7 from the first denominator, plus 2 x 5 = 10 from the second numerator, minus 3 from the third denominator, giving -7 + 10 - 3 = 0. The product inside the logarithm is therefore 2^1 x 3^0 x 5^0 = 2, and the left-hand side is log 2, exactly as required, which completes the proof.

The elegance of part (b) is that the three fractions were chosen so that the primes 3 and 5 cancel completely while the prime 2 survives with exponent exactly 1. A numerical spot-check confirms

the identity independently of the algebra: 7 log (16/15) is about 0.1962, 5 log (25/24) is about 0.0886, and 3 log (81/80) is about 0.0162, and these add to about 0.3010, which is log 2 to four decimal places. Students meeting such a 'prove that' identity should resist the temptation to evaluate each logarithm approximately and declare the two sides nearly equal; an examination proof must be exact, and the exact route is always the same sequence used here, power law outwards, product law to combine, prime factorisation to simplify, until a single logarithm stands on the left and matches the right. Note also that the identity holds for logarithms to any base, since only the laws, not the base, were used, although the course works throughout in base 10.

Conclusion

Part (a), evaluated exactly as the logarithmic-table method prescribes, gives log of the expression = 3.2933 and hence a value of approximately 1,964, agreeing with the directly computed value 1,963.79 to table accuracy, with the bar-notation handling of 0.00034 and 0.0617 being the step that decides success. Part (b) is proved exactly: the power and product laws combine the three terms into the logarithm of a single product, prime factorisation shows that product to be 2^1 x 3^0 x 5^0 = 2, and so the sum equals log 2. Together the parts show logarithms working in both directions, expansion for computation and compression for proof, and they reward the student who keeps characteristics, mantissas and signs meticulously separate at every step.

Question 3

Question 3 (20 marks): (a) If A = {1,2,3,4,5}, B = {6,7,8} and C = {3,4,6}. Verify that (A union B) intersection C = (A intersection C) union (B intersection C). (b) Insert n A.Ms between two numbers a and b.

Introduction

Part (a) of this question asks for the verification, on three concrete sets, of one of the distributive laws of set algebra, the law that intersection distributes over union. Verification here has a definite meaning: compute each side of the claimed equality separately, from the definitions of union and intersection, exhibit both resulting sets explicitly, and observe that they are identical. Part (b) asks for the companion derivation to the geometric means derivation of Question 1, this time for arithmetic means: insert n numbers between a and b so that the whole list forms an arithmetic progression, and express those numbers in terms of a, b and n. The two parts again share a method, definition first and computation second. In the set part the definitions are membership-based: an element belongs to a union if it belongs to at least one of the sets, and to an intersection if it belongs to both. In the means part the defining property of an arithmetic progression is a constant common difference, and counting how many equal steps carry us from a to b fixes that difference at once. Both parts are carried out in full below, with a membership table in the first part so that the verification can be followed element by element, and an illustrative numerical example, labelled as such, in the second.

Part (a): Computing the Left-Hand Side

First form the union A union B, the set of all elements that occur in A or in B or in both. A contributes 1, 2, 3, 4, 5 and B contributes 6, 7, 8; there is no overlap between A and B, so the union simply lists all eight numbers, A union B = {1, 2, 3, 4, 5, 6, 7, 8}. Now intersect this union with C = {3, 4, 6}, keeping only the elements that occur in both sets. Checking the elements of C one by one: 3 is in the union, 4 is in the union, and 6 is in the union, so all three survive, and no other element of the union belongs to C. The left-hand side is therefore (A union B) intersection C = {3, 4, 6}. Notice, in passing, that here the intersection has simply returned C itself, because every element of C happens to lie in A union B; that is a feature of these particular numbers, not

a step in the argument, and the right-hand side must still be computed independently for the verification to mean anything. An element-by-element membership table is given after the right-hand side computation, collecting the fate of every number from 1 to 8 under each operation, so that the equality can be audited rather than merely asserted.

Part (a): Computing the Right-Hand Side and Comparing

For the right-hand side, first form A intersection C, the elements common to A = {1, 2, 3, 4, 5} and C = {3, 4, 6}: the common elements are 3 and 4, the element 6 not being in A, so A intersection C = {3, 4}. Next form B intersection C, the elements common to B = {6, 7, 8} and C: the only common element is 6, since 7 and 8 are not in C and 3 and 4 are not in B, so B intersection C = {6}. Finally take the union of these two results: (A intersection C) union (B intersection C) = {3, 4} union {6} = {3, 4, 6}. This is exactly the set obtained for the left-hand side, element for element, so (A union B) intersection C = (A intersection C) union (B intersection C), and the distributive law stands verified for the given sets. The membership table below traces every element 1 to 8: 1, 2 and 5 appear only in A and in neither intersection; 7 and 8 appear only in B; 3 and 4 come through the A-side intersection; 6 comes through the B-side intersection; and on the left-hand route, 3, 4 and 6 are precisely the elements of the union that also lie in C, by whichever road they are reached.

Element	In A?	In B?	In C?	In (A union B) intersection C?	In (A int. C) union (B int. C)?
1	Yes	No	No	No	No

2	Yes	No	No	No	No
3	Yes	No	Yes	Yes	Yes via A
4	Yes	No	Yes	Yes	Yes via A
5	Yes	No	No	No	No
6	No	Yes	Yes	Yes	Yes via B
7	No	Yes	No	No	No
8	No	Yes	No	No	No

Understanding why the law must hold, beyond this one verification, is what turns a routine exercise into knowledge the student can reuse. An element ends up on the left-hand side exactly when it lies in C and also in at least one of A or B; it ends up on the right-hand side exactly when it lies in C together with A, or in C together with B, which is the same condition stated in other words. Because the membership conditions coincide, the sets coincide for any sets whatsoever, and the given numbers merely illustrate a universal truth. The companion law, that union distributes over intersection, (A intersection B) union C = (A union C) intersection (B union C), holds by the same style of argument and can be verified on these very sets as an exercise: on the left, A intersection B is empty, so the union with C gives {3, 4, 6}, and on the right, (A union C) intersection (B union C) = {1, 2, 3, 4, 5, 6} intersection {3, 4, 6, 7, 8} = {3, 4, 6} as well; students are encouraged to carry it through element by element in the same tabular style. Verification questions should never be answered by working one side into the other by wishful rewriting; the discipline is separate computation, explicit sets, then comparison, exactly as modelled here.

Part (b): Inserting n Arithmetic Means Between a and b

Let the n arithmetic means to be inserted be A1, A2, ..., An. Then a, A1, A2, ..., An, b is an arithmetic progression of n + 2 terms with first term a and last term b. Let d be its common difference. The terms are a, a + d, a + 2d, ..., and the k-th term is a + (k - 1) d. The last term b is the (n + 2)-th term, so b = a + (n + 2 - 1) d = a + (n + 1) d. Hence (n + 1) d = b - a, and d = (b - a) / (n + 1). Inserting n means divides the total gap between a and b into n + 1 equal steps, a point on which everything else depends and which mirrors the counting in the geometric case, where n inserted means produced n + 1 equal ratios. The means themselves are now immediate: A1 = a + d, A2 = a + 2d, and in general the k-th arithmetic mean, for k = 1, 2, ..., n, is Ak = a + k d = a + k (b - a) / (n + 1). An illustrative example, chosen by this guide only to display the method, is to insert 5 arithmetic means between 3 and 27: here d = (27 - 3) / 6 = 4, the means are 7, 11, 15, 19 and 23, and indeed 3, 7, 11, 15, 19, 23, 27 increases by 4 at each step and ends exactly at 27, verifying the formula. When n = 1 the single arithmetic mean is a + (b - a)/2 = (a + b)/2, the ordinary average of the two numbers, which is exactly why the everyday average carries the name arithmetic mean.

Conclusion

In part (a), separate computation gives (A union B) intersection C = {3, 4, 6} on the left and (A intersection C) union (B intersection C) = {3, 4} union {6} = {3, 4, 6} on the right, so the distributive law is verified for A = {1, 2, 3, 4, 5}, B = {6, 7, 8} and C = {3, 4, 6}, with the membership table tracing all eight elements through both routes. In part (b), inserting n arithmetic means between a and b gives a progression whose common difference is d = (b - a) / (n + 1), and the means are Ak = a + k (b - a) / (n + 1) for k = 1, 2, ..., n, checked on the illustrative sequence 3, 7, 11, 15, 19, 23, 27. Placed beside Question 1, the pattern of this unit

stands out clearly: n inserted means always create n + 1 equal steps, additive steps of size (b - a)/(n + 1) for arithmetic means and multiplicative steps of ratio (b/a)^(1/(n+1)) for geometric means.

Question 4

Question 4 (20 marks): Draw the graphs of: (a) y/7 = x/4 7/17. (b) y/6 = (2x)/17 + 7/4.

Introduction

This question requires the graphs of two equations, each of which is a linear equation in x and y disguised by fractions. A linear equation, once simplified, has the form y = mx + c, and its graph is always a straight line, completely determined by any two of its points; the professional method is therefore to rearrange the equation into the y = mx + c form, read off the gradient m and the intercept c, compute the two axis intercepts where the line crosses the x-axis and the y-axis, support them with a short table of values including a third point as a check, and then draw the line through those points on properly labelled axes. Both equations below are treated in exactly that way, and the graphs themselves are drawn and embedded as figures, because a graph question is answered by the drawing together with the working that fixes the line's position, not by the working alone. The fractional coefficients make the intercepts fractional as well, and that is normal; students sometimes distrust a fractional intercept and force the line through a rounder point instead, which displaces the whole graph, so the arithmetic here is shown in exact fractions first and in decimals only afterwards, so that each intercept can be plotted accurately and then checked against a third point on the same line.

Part (a): Rearranging y/7 = x/4 - 7/17

Multiply the equation y/7 = x/4 - 7/17 through by 7 to free y: y = 7x/4 - 49/17. The line therefore has gradient 7/4 = 1.75, a fairly steep positive slope, rising 7 units for every 4 units across, and its form is y = (7/4) x - 49/17. Setting x = 0 gives the y-intercept, y = -49/17, about -2.88, so the line crosses the y-axis at (0, -49/17). Setting y = 0 gives 0 = (7/4) x - 49/17, so (7/4) x = 49/17 and x = (49/17) x (4/7) = 28/17, about 1.65, so the line crosses the x-axis at (28/17, 0). A third point confirms the line: at x = 4, y = 7 - 49/17 = (119 - 49)/17 = 70/17, about 4.12, giving the point (4, 70/17); and at x = -4, y = -7 - 49/17 = -168/17, about -9.88. Plotting the two intercepts and the check point and ruling a single straight line through them gives the required graph, shown in Figure 1. Any point of the line may be tested against the original equation in its printed form; for (4, 70/17), the left side y/7 is (70/17)/7 = 10/17, and the right side x/4 - 7/17 is 1 - 7/17 = 10/17, so the point satisfies the booklet's equation exactly, confirming that the rearrangement introduced no error.

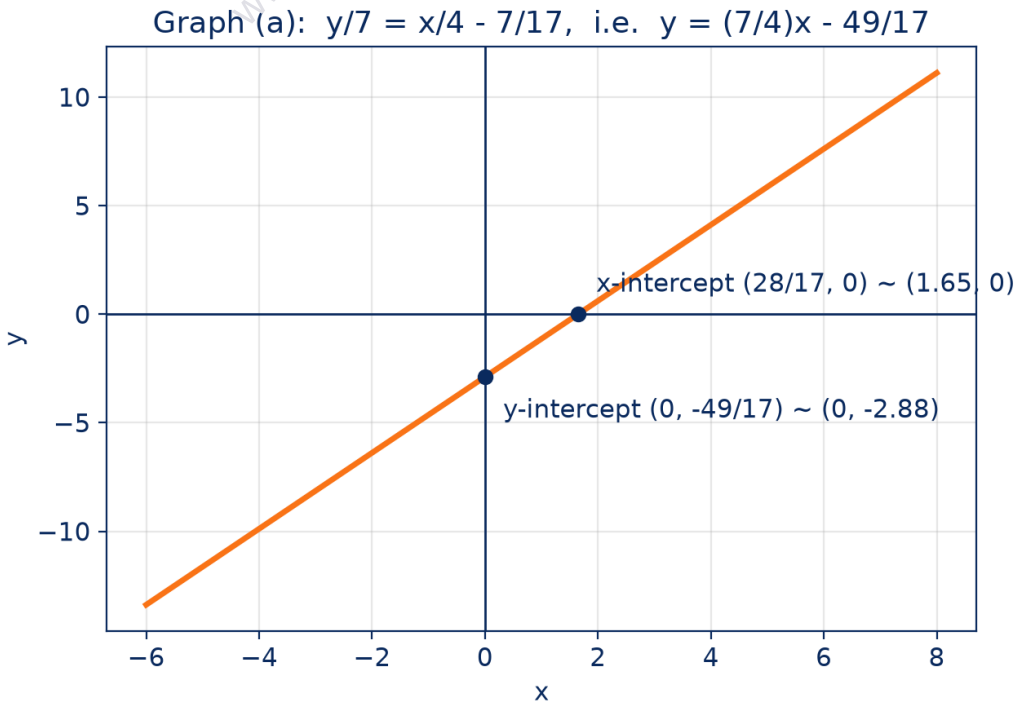


Figure 1: Graph of y/7 = x/4 - 7/17, i.e. y = (7/4)x - 49/17.

x	-4	0	28/17 (about 1.65)	4
y = (7/4)x - 49/17	-168/17 (about -9.88)	-49/17 (about -2.88)	0	70/17 (about 4.12)

Part (b): Rearranging y/6 = 2x/17 + 7/4

Multiply the equation y/6 = 2x/17 + 7/4 through by 6 to free y: y = 12x/17 + 42/4, that is y = (12/17) x + 21/2. The gradient is 12/17, about 0.71, a gentle positive slope, and the y-intercept is read off at once: setting x = 0 gives y = 21/2 = 10.5, so the line crosses the y-axis high up at (0, 21/2). Setting y = 0 gives 0 = (12/17) x + 21/2, so (12/17) x = -21/2 and x = (-21/2) x (17/12) = -119/8 = -14.875, so the line crosses the x-axis at (-119/8, 0), well to the left of the origin; the two intercepts are far apart because the line is shallow and starts high, and the axes of the drawing must be scaled to suit, which is why this graph is plotted over a wider x-range than part (a). Supporting points: at x = 17, y = 12 + 10.5 = 22.5, giving (17, 22.5); at x = -17, y = -12 + 10.5 = -1.5, giving (-17, -1.5), which sits close to the x-intercept and confirms the crossing region. The graph is shown in Figure 2. Testing (17, 22.5) in the printed equation: the left side y/6 is 22.5/6 = 3.75, and the right side 2(17)/17 + 7/4 is 2 + 1.75 = 3.75, an exact agreement, so the drawn line is the line the booklet asked for.

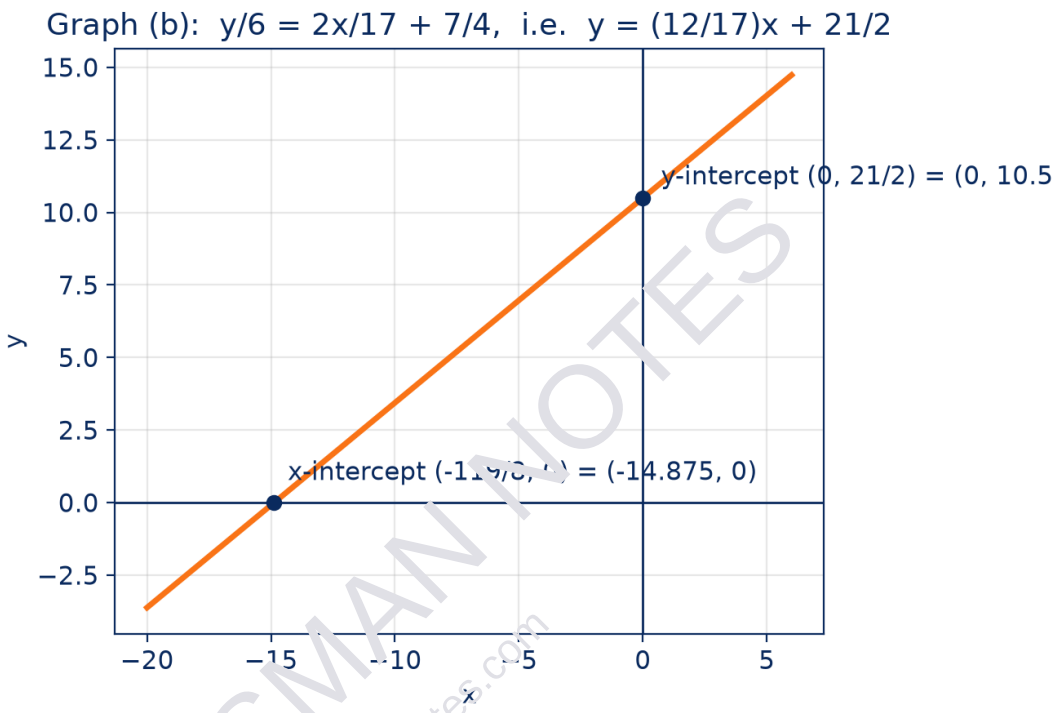


Figure 2 Graph of y/6 = 2x/17 + 7/4, i.e. y = (12/17)x + 21/2.

x	**-17**	**-119/8 (= -14.875)**	**0**	**17**
y = (12/17)x + 21/2	-1.5	0	21/2 (= 10.5)	22.5

The practical craft of drawing these graphs deserves a paragraph, because graph questions lose more marks to presentation than to algebra. First, choose scales that let both intercepts appear on the page: part (a) fits comfortably with x from -6 to 8 and y from -10 to 6, while part (b) needs x from about -20 to 6 because its x-intercept lies at -14.875; a student who uses the same small axes for both will find the second line leaving the page before it crosses the axis. Secondly, mark the intercepts as plotted points and label them with their coordinates, draw the line with a ruler

extended a little beyond the plotted points, and label the line itself with its equation. Thirdly, always carry a third point: two points determine a line, but a single arithmetic slip in one intercept then goes undetected, whereas a third point that refuses to lie on the ruled line exposes the error immediately. Finally, keep fractions exact while plotting, converting to decimals only for the physical act of measuring along the axis; -49/17 is -2.88, and plotting -2.9 is acceptable, but rounding it to -3 in the algebra and then testing points would corrupt the checks that guarantee correctness.

Conclusion

Rearranged into standard form, the two equations are y = (7/4) x - 49/17, with intercepts (28/17, 0) and (0, -49/17), gradient 1.75, and y = (12/17) x + 21/2, with intercepts (-119/8, 0) and (0, 21/2), gradient about 0.71. Their graphs are the straight lines drawn in Figures 1 and 2, each fixed by its two intercepts and confirmed by an additional point tested in the original printed equation. The method is entirely general for this unit: clear the fractions, isolate y, read off gradient and intercept, set each variable to zero in turn to find where the line meets the axes, tabulate a check point, and draw. A graph produced this way is not an artistic impression of the equation but an exact picture of it, every point of which satisfies the relation the booklet printed.

Question 5

Question 5 (20 marks): (a) Find the harmonic mean of the values 20, 500 and 1500. (b) Find the median for the set of values: 8, 6, 5, 2, 3, 1, 7, 4.

Introduction

The final question turns to descriptive statistics and to two different ideas of an average. The harmonic mean of part (a) is the right average for quantities that are rates, speeds or prices per

unit, where equal amounts of the thing measured, rather than equal times or equal counts, are involved; it is computed by averaging the reciprocals of the values and then taking the reciprocal of that average, and it is always pulled strongly towards the smallest value in the set, which the given numbers, 20, 500 and 1,500, illustrate dramatically. The median of part (b) is a positional average, the middle value of the data once they are arranged in order; it ignores the sizes of the extreme values altogether and depends only on order, and with an even count of observations it is defined as the mean of the two middle values. The two parts therefore test complementary instincts: the harmonic mean uses every value arithmetically but weights the small ones heavily, while the median uses the ordering only. Both are computed below from their definitions, with the supporting tables that a careful student would lay out, comparisons with the other familiar averages, and checks that show why each answer must lie where it does.

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Part (a): The Harmonic Mean of 20, 500 and 1,500

The harmonic mean H of n values is H = n / (1/x1 + 1/x2 + ... + 1/xn). For the three values 20, 500 and 1,500 the reciprocals are 1/20, 1/500 and 1/1500. To add them, use the common denominator 1,500: 1/20 = 75/1500, 1/500 = 3/1500 and 1/1500 = 1/1500, so the sum of the reciprocals is (75 + 3 + 1) / 1500 = 79/1500. The harmonic mean is then H = 3 / (79/1500) = 3 x 1500 / 79 = 4500/79, which is approximately 56.96. That is the answer: the harmonic mean is 4500/79, about 56.96. Observe how completely the small value 20 dominates the result: its reciprocal, 75/1500, supplies 75 of the 79 parts of the reciprocal sum, so the harmonic mean lands close to 20 even though the other two values are 500 and 1,500. This behaviour is not a defect but the very purpose of the measure: if a vehicle covers equal distances at speeds of 20, 500 and 1,500 units, its average speed is governed far more by the slow stretch, on which it

spends most of its time, than by the fast stretches it crosses in moments, and the harmonic mean captures exactly that truth.

Value x	Reciprocal 1/x	Reciprocal over denominator 1500
20	1/20	75/1500
500	1/500	3/1500
1500	1/1500	1/1500
Sum of reciprocals	79/1500	Harmonic mean = 3 / (79/1500) = 4500/79 = about 56.96

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Comparison with the other two classical means fixes the result in the student's memory and supplies a powerful check. The arithmetic mean of the three values is (20 + 500 + 1,500) / 3 = 2,020 / 3, about 673.33. The geometric mean is the cube root of their product, (20 x 500 x 1,500)^(1/3) = (15,000,000)^(1/3), about 246.62. The three means therefore stand in the order harmonic 56.96, geometric 246.62, arithmetic 673.33, illustrating the famous inequality that for positive values the harmonic mean never exceeds the geometric mean, which never exceeds the arithmetic mean, with equality only when all the values are themselves equal. Had a computation produced a harmonic mean larger than the arithmetic mean, it would certainly be wrong, and this ordering check costs nothing to apply. A second check is direct substitution: the reciprocals of the values average to (79/1500)/3 = 79/4500, and the reciprocal of 79/4500 is 4500/79, self-

consistent. Students should present the answer as the exact fraction 4500/79 with the decimal approximation beside it, since the fraction is exact and the decimal, 56.962..., is not.

Part (b): The Median of 8, 6, 5, 2, 3, 1, 7, 4

The median is found from the ordered data, so the first and indispensable step is to arrange the eight values in ascending order: 1, 2, 3, 4, 5, 6, 7, 8. There are n = 8 observations, an even number, so no single middle value exists; the middle falls between the 4th and 5th observations, which are 4 and 5 respectively. By definition the median is then the arithmetic mean of these two middle values, (4 + 5) / 2 = 9/2 = 4.5. The median of the set is therefore 4.5. The position rule in general is that the median is the average of the (n/2)-th and (n/2 + 1)-th ordered values when n is even, here the 4th and 5th, and the single ((n + 1)/2)-th ordered value when n is odd. The number-line figure below shows the ordered values with the median marked halfway between the two central ones. It is a pleasant coincidence of this data set that the ordered values form the consecutive integers 1 to 8, so the median 4.5 is also the ordinary mean of the whole set, (1 + 2 + ... + 8) / 8 = 36/8 = 4.5; that agreement is a property of evenly spaced data, not a rule, and students must not carry away the idea that median and mean generally coincide, since one outlier can drag the mean far away while the median stays exactly where the ordering puts it.

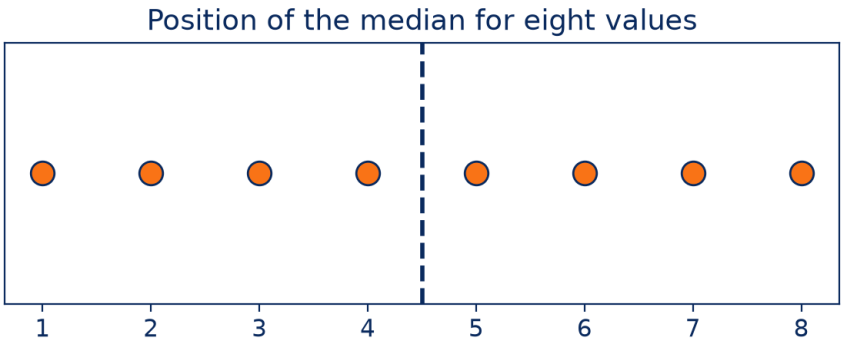


Figure 1: The ordered values 1 to 8, with the median 4.5 between the 4th and 5th values.

Position	1st	2nd	3rd	4th	5th	6th	7th	8th
Ordered value	1	2	3	4	5	6	7	8

Two common mishandlings of this very exercise are worth naming, because examiners see them every session. The first is to take the middle of the data as printed, in its original order 8, 6, 5, 2, 3, 1, 7, 4, without sorting: positions four and five of the unsorted list are 2 and 3, whose mean 2.5 is then wrongly offered as the median, whereas sorting is not an optional cosmetic step but the definition of the median itself. The second is to stop at the two middle values and report '4 and 5' as the answer when n is even; a median must be a single number, and the defining convention averages the pair. The robustness of the median is the reason statisticians prefer it for incomes, house prices and other skewed data: if the largest value in this set were replaced by 800 instead of 8, the mean would leap from 4.5 to about 103.5, while the median would remain 4.5 untouched, still describing the centre of the bulk of the data faithfully. Together with the mode, the value occurring most frequently, which this set does not possess since every value occurs once, the median completes the trio of averages examined in this unit, each answering a different question about a data set.

Conclusion

In part (a), the harmonic mean of 20, 500 and 1,500 is 3 divided by the sum of the reciprocals, 3 / (79/1500) = 4500/79, approximately 56.96, a result dominated by the smallest value exactly as the theory of the harmonic mean predicts, and sitting correctly below the geometric mean of about 246.62 and the arithmetic mean of about 673.33. In part (b), the ordered data 1, 2, 3, 4, 5,

6, 7, 8 have eight values, so the median is the mean of the 4th and 5th values, (4 + 5) / 2 = 4.5. The two answers, 56.96 approximately and 4.5 exactly, close the assignment with the unit's central statistical lesson: choose the average that matches the structure of the problem, rates and reciprocals calling for the harmonic mean, position and order calling for the median, and carry out each from its definition rather than from habit.

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