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Semester: Autumn 2026

ALLAMA IQBAL OPEN UNIVERSITY, ISLAMABAD

Course: Business Mathematics (1429) • Level: B.A, B.Com

Semester: Autumn 2026

Assignment No. 1 (Units 1–4)

SOLVED ASSIGNMENT GUIDE

(For Study & Reference)

Note: This guide is prepared for study and reference purposes. Students are advised to read the relevant units of the course book, understand the concepts and methods, and then write the assignment in their own words. Total marks of the assignment are 100 (pass marks 40); each question carries the marks shown with it. All working is shown step by step so that the method can be followed and reproduced.

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Question 1: Probability Theory (20 Marks)

a) Find the sample space for choosing an odd number from 1 to 15 at random. b) What is the difference between mutually exclusive events and collectively exhaustive events? c) In a game show, the contestant is shown 10 boxes, 3 of which contain prizes. If a contestant is allowed to select any three boxes, then what is the probability that (I) the contestant wins all three prizes, and (II) only one selected box contains a prize? d) The probability that an applicant for pilot school will be admitted is 0.5. If three applicants are selected at random, what is the probability that (i) all three will be admitted, (ii) none will be admitted, and (iii) only one will be admitted?

Introduction

Probability is the branch of mathematics that measures uncertainty. Whenever the result of an action cannot be predicted with complete certainty in advance — drawing a number, opening a box, or applying for admission — we describe the situation as a random experiment, list everything that can possibly happen, and then attach a number between 0 and 1 to each possibility. That number, the probability, tells us how likely an outcome is: 0 means the outcome is impossible, 1 means it is certain, and values in between measure the degree of likelihood. This question takes us through the foundations of the subject in four steps. Part (a) asks for a sample space, the complete list of possible outcomes. Part (b) asks for two ideas that are constantly confused with each other, mutually exclusive events and collectively exhaustive events. Part (c) is a counting problem in which the probability is found by combinations, because the contestant chooses boxes and the order of choosing does not matter. Part (d) is a repeated-trials problem in which the multiplication rule for independent events does the work. Together the four parts show the three habits that solve almost every elementary probability problem: list the possibilities carefully, count them correctly, and only then divide favourable cases by total cases.

Part (a): Sample Space for Choosing an Odd Number from 1 to 15

The sample space, usually written S, is the set of all possible outcomes of a random experiment. Here the experiment is to choose one number at random, with the condition that attention is restricted to the odd numbers from 1 to 15. The odd numbers in this range are 1, 3, 5, 7, 9, 11, 13 and 15, so the sample space is S = {1, 3, 5, 7, 9, 11, 13, 15}. There are eight outcomes in the sample space, so n(S) = 8. Because the choice is made at random, every one of these eight

outcomes is equally likely, and therefore the probability of any single one of them, for example the number 7, is 1/8 = 0.125. A useful check is that the probabilities of all the outcomes in a sample space must add up to 1, and here 8 × (1/8) = 1, as required. It is worth noticing why the even numbers do not appear: an experiment is defined by what the experimenter is doing and observing. If the experiment had been 'choose any number from 1 to 15', the sample space would have contained fifteen outcomes; because the choice is restricted to odd numbers, the sample space contains only the eight odd ones. Writing the sample space correctly at the start is not a formality — almost every mistake in probability begins with a wrong or incomplete sample space, and almost every correct solution begins by listing it exactly as we have done here.

Part (b): Mutually Exclusive Events versus Collectively Exhaustive Events

Two events are called mutually exclusive (or disjoint) when they cannot happen together in a single trial of the experiment: the occurrence of one excludes the occurrence of the other. In set language, their intersection is empty, $A \cap B = \emptyset$, and consequently the addition rule takes its simplest form, P(A or B) = P(A) + P(B). When a single die is thrown, for example, the event 'an even number appears' and the event 'an odd number appears' are mutually exclusive, because one throw cannot show a number that is both even and odd. Similarly, in drawing one card, 'the card is a king' and 'the card is a queen' are mutually exclusive. Events are called collectively exhaustive, on the other hand, when together they cover the whole sample space: at least one of them is certain to occur, because no possible outcome lies outside all of them. In set language their union is the sample space itself, and the sum of their probabilities is 1 for a

complete exhaustive list of mutually exclusive outcomes. On a die, the events '1 or 2', '3 or 4' and '5 or 6' are collectively exhaustive because every possible throw belongs to one of them.

The difference between the two ideas is therefore a difference between two different questions. Mutual exclusiveness is about overlap: it asks whether two events can occur together, and the answer is no. Collective exhaustiveness is about coverage: it asks whether the events together leave any outcome unaccounted for, and the answer is no. One idea concerns the intersection of events, the other concerns their union. The two properties are independent of each other. Events may be mutually exclusive without being exhaustive — 'king' and 'queen' in one draw are exclusive, but a jack is neither, so they do not exhaust the pack. Events may be exhaustive without being exclusive — 'the card is red' and 'the card is a face card' together with 'the card is black and not a face card' can cover the pack while still overlapping. And events may be both at once, which is the most useful case of all: the even and odd outcomes of a die are both mutually exclusive and collectively exhaustive, forming what is called a partition of the sample space. Recognising which property a problem is using matters because the rules differ: exclusiveness lets us add probabilities directly, while exhaustiveness tells us the probabilities must total 1, a fact used constantly to find a missing probability by subtracting the others from 1.

Part (c): The Game Show — Choosing Three Boxes out of Ten

There are 10 boxes, of which 3 contain prizes and 7 are empty, and the contestant selects any 3 boxes. Because the order in which the boxes are picked does not matter, the total number of equally likely selections is the number of combinations of 10 objects taken 3 at a time: C(10, 3)

= 10! / (3! × 7!) = (10 × 9 × 8) / (3 × 2 × 1) = 120. Every probability in this part is therefore a count of favourable selections divided by 120. For part (I), winning all three prizes, the contestant must pick exactly the 3 prize boxes and nothing else. There is only C(3, 3) × C(7, 0) = 1 × 1 = 1 such selection, so the required probability is 1/120 ≈ 0.0083, a little less than one chance in a hundred — which matches our intuition that sweeping all three prizes with only three picks is a very lucky outcome. For part (II), exactly one selected box containing a prize, the contestant must choose 1 of the 3 prize boxes and 2 of the 7 empty boxes. The number of such selections is C(3, 1) × C(7, 2) = 3 × 21 = 63, so the probability is 63/120 = 0.525. In other words, the contestant is slightly more likely than not to walk away with exactly one prize. The pattern of the method is worth stating generally, because it solves the whole family of 'drawing without replacement' problems: split the population into the wanted group and the unwanted group, choose the required number from each group by combinations, multiply the two counts, and divide by the total number of ways of choosing from the whole population.

A good way to check work of this kind is to complete the distribution. The probability of no prize at all is C(7, 3)/120 = 35/120 ≈ 0.2917, the probability of exactly two prizes is C(3, 2) × C(7, 1)/120 = 21/120 = 0.175, and adding all four cases gives (35 + 63 + 21 + 1)/120 = 120/120 = 1, exactly as a complete set of mutually exclusive and collectively exhaustive cases must. This check is not an extra decoration; it is the practical use of Part (b) of this very question, and students are advised to use it whenever a distribution has been built case by case.

Part (d): Admission to Pilot School — Three Independent Applicants

For one applicant, the probability of admission is p = 0.5 and the probability of rejection is q = 1 − 0.5 = 0.5. The three applicants are selected at random and their results do not influence one another, so the events are independent and probabilities of combined results are found by multiplication. (i) All three admitted: the required probability is 0.5 × 0.5 × 0.5 = 0.125. (ii) None admitted: each applicant is rejected with probability 0.5, so the probability is again 0.5 × 0.5 × 0.5 = 0.125. The equality of these two answers is a special feature of p = 0.5: admission and rejection are symmetric, so 'all succeed' and 'all fail' must be equally likely. (iii) Only one admitted: here we must be careful, because there are three different ways this can happen — the first applicant admitted and the other two rejected, the second admitted and the others rejected, or the third admitted and the others rejected. Each single pattern has probability 0.5 × 0.5 × 0.5 = 0.125, and the patterns are mutually exclusive, so the required probability is 3 × 0.125 = 0.375. In the language of the binomial distribution this is $C(3, 1) \times (0.5)^1 \times (0.5)^2 = 0.375$. Once more the whole picture can be checked: exactly two admitted also has probability 3 × 0.125 = 0.375, and 0.125 + 0.375 + 0.375 + 0.125 = 1, confirming that the four cases exhaust all possibilities.

Conclusion

To conclude: the sample space in (a) is S = {1, 3, 5, 7, 9, 11, 13, 15} with eight equally likely outcomes. Mutually exclusive events cannot occur together, while collectively exhaustive events together cover every possible outcome; the first idea governs the intersection of events, the second their union, and the two should never be confused. In the game show, the probability of winning all three prizes is 1/120 ≈ 0.0083 and the probability of winning exactly

one prize is 63/120 = 0.525. For the pilot school applicants, the probabilities are 0.125 that all three are admitted, 0.125 that none is admitted, and 0.375 that exactly one is admitted. In every part the same discipline produced the answer: define the experiment, list or count the possibilities honestly, apply the addition rule only where exclusiveness holds and the multiplication rule only where independence holds, and finally check that the completed set of probabilities sums to one.

Question 2: Random Variables (20 Marks)

a) Let X be a random variable having normal distribution with mean 48 and standard deviation 10. Then find P(X ≤ 41). b) Define a random variable. What is the difference between a discrete random variable and a continuous random variable? c) Construct the discrete probability distribution that corresponds to the experiment of tossing a fair coin three times. Suppose the random variable X equals the number of heads occurring in three tosses. What is the probability of two or more heads? d) Compute the mean, median, mode and standard deviation for the following distribution — Value of X: 20, 30, 40, 50, 60; Frequency: 8, 12, 10, 12, 8.

Introduction

A random variable is the bridge between a random experiment and arithmetic. Experiments produce descriptions — heads and tails, admitted and rejected, passed and failed — but calculations need numbers. A random variable attaches a number to every outcome, and once that is done the whole machinery of averages, spread and standard tables can be brought to bear on chance itself. This question works through the idea from four directions. Part (a) uses

the most important continuous distribution of all, the normal distribution, and shows how a probability question about X is converted into a question about the standard normal variable Z and answered from tables. Part (b) asks for the definition and for the distinction between discrete and continuous variables, the distinction that decides which tools a problem needs. Part (c) builds a complete discrete distribution from scratch for three tosses of a fair coin — the same experiment that appeared in Question 1, now counted by number of heads. Part (d) is a calculation exercise on grouped data: from a small frequency distribution the mean, median, mode and standard deviation are all extracted, and, as we shall see, they agree with each other in an instructive way because the data are symmetric.

Part (a): A Normal Probability — P(X ≤ 41) when Mean = 48 and Standard Deviation = 10

The normal distribution is the continuous, bell-shaped distribution described by its mean μ, which fixes its centre, and its standard deviation σ, which fixes its spread. Here μ = 48 and σ = 10. No table can list every normal distribution, so every normal problem is first converted into the standard normal distribution, whose mean is 0 and standard deviation is 1, by the formula Z = (X − μ)/σ. The value X = 41 lies 7 units below the mean, and in standard units this is Z = (41 − 48)/10 = −7/10 = −0.70. The required probability is therefore P(X ≤ 41) = P(Z ≤ −0.70). By the symmetry of the bell curve about its centre, the area to the left of −0.70 equals the area to the right of +0.70, and the standard table gives the area between the mean and Z = 0.70 as 0.2580, so the area to the left of −0.70 is 0.5000 − 0.2580 = 0.2420. Hence P(X ≤ 41) ≈ 0.2420, that is, about 24.2 per cent. The answer is sensible: 41 is below the mean, so the probability of falling at or below it must be less than one half, but it is only seven-tenths of a standard deviation

below the mean, so the probability should still be a substantial fraction and not a tiny tail value — 0.2420 fits both expectations. Students should note the three steps, because they never change: draw or imagine the curve and shade the required region, convert the boundary value to a Z-score, and read the area from the table, adding or subtracting 0.5 according to which side of the mean the region lies on. The shaded region is shown in Figure 1.

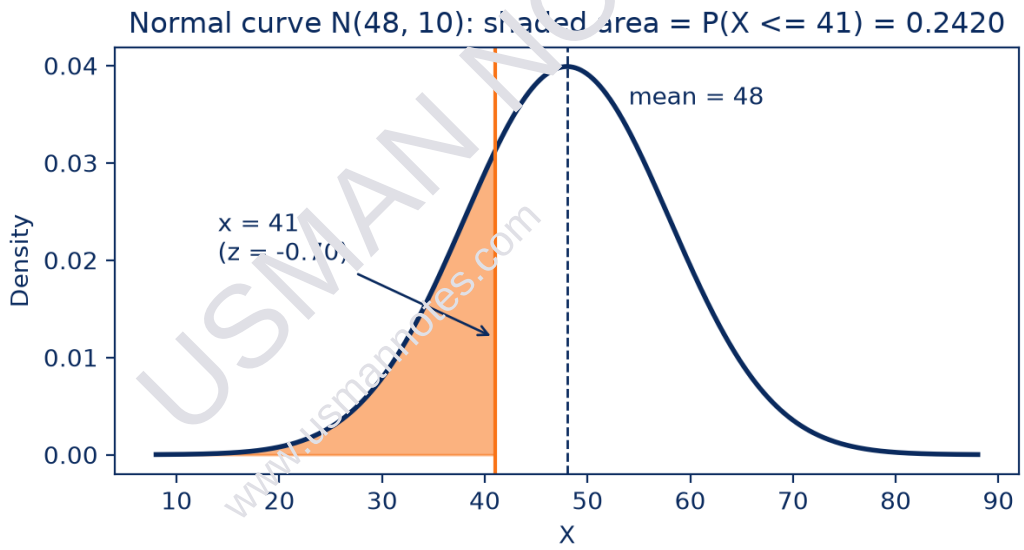


Figure 1: The normal curve with mean 48 and standard deviation 10; the shaded area to the left of X = 41 is 0.2420.

Part (b): What Is a Random Variable? Discrete versus Continuous

A random variable is a rule that assigns a numerical value to each outcome of a random experiment. If the experiment is tossing a coin three times, the rule 'count the heads' assigns 0 to TTT, 1 to HTT, THT and TTH, 2 to HHT, HTH and THH, and 3 to HHH. If the experiment is testing a bulb, the rule 'record its life in hours' assigns a measurement to each bulb. The word 'random' records that the value is not known until the experiment is performed; the word

'variable' records that the value varies from trial to trial. Random variables are conventionally written with capital letters, X, Y, Z, and particular values with small letters. A discrete random variable is one whose possible values can be listed and counted — they are separated from one another, with gaps in between, even when the list is endless, as with the number of trials needed for the first success. A continuous random variable is one whose possible values fill an interval without gaps: between any two possible values a third value is always possible, as with height, weight, time, temperature and the life of a bulb. The difference is not a piece of pedantry, because the probability machinery is different in the two cases. For a discrete variable we prepare a probability distribution, a table pairing each value x with its probability P(X = x), where every probability lies between 0 and 1 and the probabilities sum to 1; the probability of a single named value is usually positive and meaningful. For a continuous variable, probabilities are areas under a curve, the probability of any single exact value is zero, and only statements about intervals, such as P(X ≤ 41) in Part (a), have positive answers. Counting problems therefore signal a discrete variable and measurement problems a continuous one, and recognising the type at the outset tells the student immediately whether to build a table or to reach for the normal curve and its Z-table.

Part (c): Distribution of the Number of Heads in Three Tosses

Tossing a fair coin three times gives $2^3 = 8$ equally likely outcomes: HHH, HHT, HTH, THH, HTT, THT, TTH and TTT, each with probability 1/8. Let X be the number of heads. Counting heads outcome by outcome, X = 3 occurs once (HHH), X = 2 occurs three times (HHT, HTH, THH), X = 1 occurs three times (HTT, THT, TTH) and X = 0 occurs once (TTT). Dividing each count by 8 gives the discrete probability distribution shown in Table 1: P(X = 0) = 1/8 = 0.125, P(X = 1) = 3/8 =

0.375, P(X = 2) = 3/8 = 0.375 and P(X = 3) = 1/8 = 0.125. The probabilities sum to 0.125 + 0.375 + 0.375 + 0.125 = 1, as every complete distribution must. The symmetry of the table is no accident: a fair coin treats heads and tails alike, so the count of heads must be distributed symmetrically about 1.5. The question then asks for the probability of two or more heads, P(X ≥ 2). Adding the two relevant rows, P(X ≥ 2) = P(X = 2) + P(X = 3) = 3/8 + 1/8 = 4/8 = 0.5. So in three tosses of a fair coin, a majority of heads is exactly an even-money event — a neat result that could also have been guessed from symmetry, since 'two or more heads' and 'two or more tails' are equally likely, cannot both happen, and between them cover everything except... in fact they do cover everything here, because with three tosses one side must have a majority. Figure 2 displays the distribution: the two orange bars are the event whose probability is 0.5.

Table 1: Probability distribution of the number of heads in three tosses of a fair coin

X = number of heads	0	1	2	3	Total
Number of outcomes	1	3	3	1	8
P(X = x)	0.125	0.375	0.375	0.125	1.000

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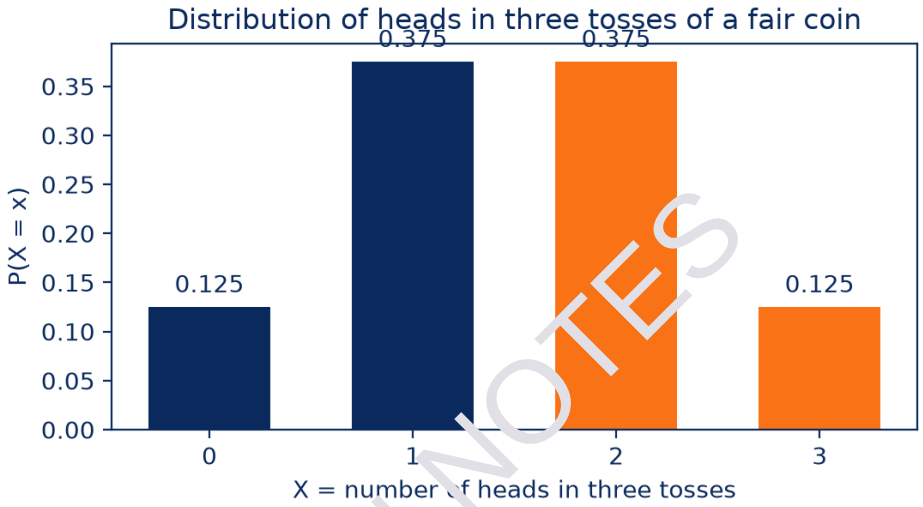


Figure 2: The distribution of X; the orange bars (X = 2 and X = 3) together have probability 0.5.

Part (d): Mean, Median, Mode and Standard Deviation of the Given Distribution

The data consist of the values 20, 30, 40, 50 and 60 with frequencies 8, 12, 10, 12 and 8 respectively, so the total frequency is N = 8 + 12 + 10 + 12 + 8 = 50. (The booklet calls this a discrete probability distribution; dividing each frequency by 50 gives the corresponding probabilities 0.16, 0.24, 0.20, 0.24 and 0.16, and every result below is the same whether the calculation is done with frequencies or with these probabilities, as the working shows.) The mean is the frequency-weighted average, mean = ΣfX / N. The products fX are 8 × 20 = 160, 12 × 30 = 360, 10 × 40 = 400, 12 × 50 = 600 and 8 × 60 = 480, giving ΣfX = 2000, and therefore the mean is 2000/50 = 40. For the median, arrange the fifty observations in order and take the average of the 25th and 26th values. The cumulative frequencies are 8 (up to 20), 20 (up to 30), 30 (up to 40), 42 (up to 50) and 50 (up to 60), so positions 21 to 30 all carry the value 40; both the 25th and the 26th observations are therefore 40, and the median is 40. The mode is the most frequent value; here the largest frequency, 12, occurs twice, at X = 30 and at X = 50, so

the distribution is bimodal with modes 30 and 50. Finally, the standard deviation measures the typical distance of the values from the mean. The squared deviations from 40, weighted by frequency, are $8 \times (20 - 40)^2 = 8 \times 400 = 3200$, $12 \times (30 - 40)^2 = 12 \times 100 = 1200$, $10 \times 0 = 0$, $12 \times (50 - 40)^2 = 1200$ and $8 \times (60 - 40)^2 = 3200$, a total of $\Sigma f(X - \text{mean})^2 = 8800$. The variance is $8800/50 = 176$, and the standard deviation is $\sqrt{176} \approx 13.27$.

Table 2: Working table for the mean, median and standard deviation

Value of X	Frequency f	fX	Cumulative frequency	$f(X - 40)^2$
20	8	160	8	3,200
30	12	360	20	1,200
40	10	400	30	0
50	12	600	42	1,200
60	8	480	50	3,200
Total	50	2,000	—	8,800

The three averages tell a consistent story. Mean and median coincide at 40, the exact centre of the values, which is what perfectly symmetric data must produce, and the frequencies themselves — 8, 12, 10, 12, 8 — are visibly symmetric about the central value. The modes sit symmetrically at 30 and 50, one step either side of the centre. The standard deviation of about 13.27 should be read against the range of the data, 20 to 60: a typical observation lies roughly 13 units from the centre, which for values spread over a 40-unit range is a moderate, believable amount of spread. A student who obtained, say, a standard deviation of 40 or of 1.3 should suspect an arithmetical slip, because neither figure is compatible with the data actually given — one reason why interpreting a result, and not merely producing it, is a part of the calculation.

Conclusion

In summary: for the normal variable of Part (a), standardising X = 41 gives Z = −0.70 and P(X ≤ 41) ≈ 0.2420. A random variable is a numerical rule attached to the outcomes of a random experiment; it is discrete when its values can be listed and counted, each carrying its own probability in a table summing to 1, and continuous when its values fill an interval, so that probabilities are areas under a curve and single values have probability zero. Three tosses of a fair coin give the distribution P(0) = 0.125, P(1) = 0.375, P(2) = 0.375, P(3) = 0.125, and the probability of two or more heads is exactly 0.5. For the frequency distribution of Part (d) the mean is 40, the median is 40, the modes are 30 and 50 (the distribution is bimodal), and the standard deviation is √176 ≈ 13.27. The lesson running through the question is that a random variable turns chance into numbers, and once the type of variable and its distribution are correctly identified, the required probability or average is a matter of careful, checkable arithmetic.

Question 3: First-Degree Equations (20 Marks)

Solve the following first-degree equations: a. x − 56 = 50x + 30. b. −15 + 3x = 24x − 3. c. (4x + 96) − (−16 + 40x) + 45 = 0.

Introduction

A first-degree equation, also called a linear equation in one unknown, is an equation in which the unknown appears only to the first power — there is no x^2, no 1/x and no square root of x. Its standard form is ax + b = 0 with a ≠ 0, and it always has exactly one solution. Such equations are the workhorse of business mathematics because so many commercial relationships are

linear: a total cost built from a fixed charge plus a charge per unit, a price reduced by a fixed discount, or a quantity split in a stated ratio all lead directly to a first-degree equation. The method of solution rests on one principle, the balance principle: an equation is like a balance in equilibrium, and whatever is done to one side must be done to the other side if the equality is to survive. Adding the same number to both sides, subtracting the same number from both sides, and multiplying or dividing both sides by the same non-zero number never change the solution. The practical strategy is always the same: first remove brackets and simplify each side, then collect all terms containing the unknown on one side and all constant terms on the other, and finally divide by the coefficient of the unknown. Every answer should then be verified by substitution, putting the value found back into the original equation and confirming that the left-hand side and the right-hand side really are equal. In this question the three equations are deliberately of increasing awkwardness: part (a) has the unknown on both sides with the larger coefficient on the right, part (b) contains negative constants that trap the careless, and part (c) hides the unknown inside brackets with a subtraction sign in front of the second bracket — the single most common source of error in elementary algebra. Each part below is solved step by step, and each answer is checked by substitution, as the verification habit is worth as much as any single answer.

Part (a): x – 56 = 50x + 30

The unknown appears on both sides, and the coefficient on the right (50) is larger than the coefficient on the left (1). We collect the unknowns on one side and the constants on the other. Subtract x from both sides — or, equivalently, bring the x-terms to the left and the constants to the right, changing the sign of every term that crosses the equality sign. Moving the terms

gives: x − 50x = 30 + 56. Simplifying the left side, (1 − 50)x = −49x, and the right side is 86, so the equation becomes −49x = 86. Dividing both sides by −49 gives x = 86 / (−49) = −86/49 ≈ −1.76. The negative answer should not alarm us: an equation knows nothing about the story that produced it, and a fractional, negative solution is perfectly respectable. Check by substitution: the left-hand side is x − 56 = −86/49 − 56 = −86/49 − 2744/49 = −2830/49 ≈ −57.76. The right-hand side is 50x + 30 = 50 × (−86/49) + 30 = −4300/49 + 1470/49 = −2830/49 ≈ −57.76. The two sides agree exactly, so x = −86/49 is correct. Notice the general lesson: when the coefficient of x turns out negative after collection, do not try to avoid the minus sign by moving terms a second time; simply divide through by the negative coefficient carefully. The majority of errors in this type of equation are sign errors made while transposing terms, and the substitution check is the reliable protection against them, since a wrong sign almost never survives being put back into the original equation.

Part (b): −15 + 3x = 24x − 3

Again the unknown is on both sides. Collect the x-terms on one side: subtract 3x from both sides, or transpose, so that −15 + 3 − ... — working systematically: bringing the x-terms to the left and the constants to the right gives 3x − 24x = −3 + 15. The left side simplifies to −21x and the right side to 12, so −21x = 12 and x = 12/(−21) = −12/21 = −4/7 ≈ −0.57. The fraction has been reduced to its lowest terms by dividing numerator and denominator by 3, as final answers should always be presented in simplest form. Check by substitution: the left-hand side is −15 + 3x = −15 + 3 × (−4/7) = −15 − 12/7 = −105/7 − 12/7 = −117/7 ≈ −16.71. The right-hand side is 24x − 3 = 24 × (−4/7) − 3 = −96/7 − 21/7 = −117/7 ≈ −16.71. The sides agree, so x = −4/7 is the solution. Two features of this equation deserve comment. First, the constant −15 stands at the

head of the left side; when transposing, the whole constant, together with its sign, moves — students who move only the 15 and drop the minus sign obtain a wrong answer that a check would immediately expose. Second, the equation could equally have been solved by collecting the unknowns on the right instead: −15 + 3 = 24x − 3x gives −12 = 21x and x = −12/21 = −4/7, the same result. There is no rule about which side must carry the unknown; there is only the requirement of consistency, and choosing the side that keeps the coefficient of x positive is a legitimate convenience, not a necessity.

Part (c): (4x + 96) − (−16 + 40x) + 45 = 0

This equation must be simplified before anything is transposed, because the unknown is locked inside brackets. The first bracket is preceded by an understood plus sign, so it is removed term by term unchanged: 4x + 96. The second bracket is preceded by a minus sign, and removing it reverses the sign of every term inside: −(−16 + 40x) = +16 − 40x. This step is the heart of the question: 'minus a negative is a plus' turns −16 into +16, while 'minus a positive is a minus' turns +40x into −40x. The equation therefore becomes 4x + 96 + 16 − 40x + 45 = 0. Now collect like terms. The x-terms give 4x − 40x = −36x, and the constants give 96 + 16 + 45 = 157, so the equation reduces to −36x + 157 = 0, that is, 36x = 157 and x = 157/36 ≈ 4.36. The fraction 157/36 does not simplify, since 157 is a prime number not dividing 36. Check by substitution into the original, bracketed equation: inside the first bracket, 4x + 96 = 4 × (157/36) + 96 = 157/9 + 96 ≈ 17.44 + 96 = 113.44. Inside the second bracket, −16 + 40x = −16 + 40 × (157/36) = −16 + 1570/9 ≈ −16 + 174.44 = 158.44. The left-hand side is therefore 113.44 − 158.44 + 45 = 0, exactly as required. The answer x = 157/36 ≈ 4.36 is confirmed. Worked in fractions throughout,

the check is exact: 157/9 + 96 = 1021/9, and −16 + 1570/9 = 1426/9, and 1021/9 − 1426/9 + 45 = −405/9 + 45 = −45 + 45 = 0.

Conclusion

The three solutions are: (a) x = −86/49 ≈ −1.76; (b) x = −4/7 ≈ −0.57; and (c) x = 157/36 ≈ 4.36. Each has been verified by substitution into the original equation, which is the only completely reliable test. The question illustrates the whole technique of first-degree equations in miniature: remove brackets first, remembering that a minus sign before a bracket changes every sign inside it; collect unknowns on one side and constants on the other, changing signs as terms cross; divide by the coefficient of the unknown; present fractions in lowest terms; and check. Notice also that none of the three answers is a whole number, and two are negative. Textbook exercises are often rigged to give neat answers, but real business equations are not, and a student who distrusts a fractional or negative answer merely because it is untidy will change correct work into wrong work. The equation, solved honestly and checked by substitution, is the authority — not the appearance of the answer.

Question 4: Quadratic Equations and a Rectangle Problem (20 Marks)

a) Solve the following quadratic equations using the quadratic formula: (i) $5x^2 + 4x - 1 = 0$; (ii) $4t^2 - 4 = 0$. b) A railing is to enclose a rectangular area of 1600 square feet. The length of the plot is twice the width. How much railing must be used?

Introduction

A quadratic equation is an equation in which the highest power of the unknown is two; its standard form is $ax^2 + bx + c = 0$ with $a \neq 0$. Where a first-degree equation has exactly one solution, a quadratic generally has two, because the graph of a quadratic — a parabola — can meet the x-axis in two points. Quadratics arise naturally in business and everyday problems whenever an area, a product of two quantities, or a profit built from price times quantity is involved, and Part (b) of this question is exactly such a case: the area condition on a rectangular plot produces a quadratic for its width. Part (a), however, specifies the method: the equations are to be solved using the quadratic formula, $x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$. The formula is completely general — it solves every quadratic, whether or not the equation factorises — and the quantity under the root sign, $D = b^2 - 4ac$, called the discriminant, even predicts the character of the roots before they are found: two distinct real roots when $D > 0$, two equal roots when $D = 0$, and no real roots when $D < 0$. The discipline in using the formula is to identify a, b and c first, with their signs, straight from the standard form, and to substitute carefully. Part (b) then shows the other half of the skill: translating words into an equation, solving it, and — because lengths cannot be negative — judging which of the two mathematical roots the real-world problem accepts, before answering the question actually asked, which is not the width but the amount of railing, that is, the perimeter.

Part (a)(i): $5x^2 + 4x - 1 = 0$ by the Quadratic Formula

Comparing with $ax^2 + bx + c = 0$, we have $a = 5$, $b = 4$ and $c = -1$; note in particular that c carries its negative sign into the formula. The discriminant is $D = b^2 - 4ac = 4^2 - 4(5)(-1) = 16 + 20 = 36$, which is positive, so two distinct real roots exist; indeed 36 is a perfect square, so the roots will

be rational. Substituting: x = [−4 ± √36] / (2 × 5) = [−4 ± 6] / 10. Taking the plus sign: x = (−4 + 6)/10 = 2/10 = 1/5 = 0.2. Taking the minus sign: x = (−4 − 6)/10 = −10/10 = −1. The roots are therefore x = 0.2 and x = −1. Check the first root: $5(0.2)^2 + 4(0.2) - 1 = 5(0.04) + 0.8 - 1 = 0.2 + 0.8 - 1 = 0$ ✓ Check the second: $5(-1)^2 + 4(-1) - 1 = 5 - 4 - 1 = 0$ ✓ Both roots satisfy the equation. It is instructive that this equation also factorises, as (5x − 1)(x + 1) = 0, giving the same roots; but the question prescribes the formula, and the formula has the great merit of working even when, as in most practical problems, no factorisation exists. The commonest error in this routine is the handling of −4ac when c is negative: −4(5)(−1) is +20, not −20, and a student who writes D = 16 − 20 = −4 will wrongly report that the equation has no real roots. Writing a, b and c down explicitly, with signs, before any substitution is the simple safeguard, and checking the discriminant's sign against expectations is the second one.

Part (a)(ii): $4t^2 - 4 = 0$ by the Quadratic Formula

Here the equation contains no term in t, which simply means that in standard form a = 4, b = 0 and c = −4; a missing term is a term with coefficient zero, not an excuse to skip the formula. The discriminant is $D = b^2 - 4ac = 0^2 - 4(4)(-4) = 0 + 64 = 64$, positive, so there are two distinct real roots. Substituting: t = [−0 ± √64] / (2 × 4) = ±8/8 = ±1. The roots are t = 1 and t = −1. Check: $4(1)^2 - 4 = 0$ ✓and $4(-1)^2 - 4 = 0$ ✓, since squaring removes the sign. Of course this equation can also be solved directly — $4t^2 = 4$ gives $t^2 = 1$ and t = ±1 — and the direct method is a useful confirmation, but the formula reaches the same answer and would continue to work if the equation were less tidy, for instance $4t^2 + 3t - 4 = 0$. The symmetrical pair of roots ±1 is characteristic of quadratics with b = 0, whose parabolas are symmetric about the vertical axis. The lesson generalises: whenever the linear term is absent, expect roots equal in size and

opposite in sign; whenever the constant term is absent, expect one root to be zero. Reading such expectations off the equation before solving gives the student a prediction to test the computed roots against, and a mismatch is an early warning of a substitution error.

Part (b): The Rectangular Plot of 1600 Square Feet

Let the width of the plot be w feet. The length is twice the width, so the length is 2w feet. The area of a rectangle is length × width, and the area is given as 1600 square feet, so the condition of the problem is 2w × w = 1600, that is, $2w^2 = 1600$, or $w^2 = 800$. Taking square roots, w = ±√800 = ±20√2 ≈ ±28.28 feet. Mathematics offers two roots, but a width is a length and cannot be negative, so the negative root is rejected on physical grounds and w = 20√2 ≈ 28.28 feet. The length is twice this: 40√2 ≈ 56.57 feet. As a check, length × width = 40√2 × 20√2 = 800 × 2 = 1600 square feet, exactly the required area ✓. The question, however, does not ask for the width or the length; it asks how much railing must be used, and railing is needed all the way round the plot, so the quantity required is the perimeter: P = 2(length + width) = 2(40√2 + 20√2) = 2 × 60√2 = 120√2 ≈ 169.71 feet. Approximately 170 feet of railing must therefore be used. Two habits are on display in this short problem and both are worth copying. The first is to answer the question that was asked: many scripts would stop at the width and lose the final marks, because the examiner asked for railing, not for dimensions. The second is the rejection of the negative root, with the reason stated. In pure algebra both roots of $w^2 = 800$ are correct; in an applied problem the context is part of the mathematics, and a solution that reports a width of '−28.28 feet' without comment shows that the answer was never thought about. Stating 'the negative root is impossible for a length' converts a bare calculation into a solved problem.

Conclusion

To conclude: by the quadratic formula, $5x^2 + 4x - 1 = 0$ has roots $x = 1/5$ and $x = -1$ (discriminant 36), and $4t^2 - 4 = 0$ has roots $t = 1$ and $t = -1$ (discriminant 64); both pairs were verified by substitution, and in each case the discriminant correctly predicted two distinct real roots. For the plot, setting the width as w gives $2w^2 = 1600$, so $w = 20\sqrt{2} \approx 28.28$ feet and the length is $40\sqrt{2} \approx 56.57$ feet; the railing required is the per meter, $120\sqrt{2} \approx 169.71$ feet, about 170 feet. The question as a whole teaches the complete quadratic routine: put the equation in standard form, record a, b and c with their signs, compute the discriminant and note what it promises, apply the formula without dropping the ± sign, check the roots, and in applied problems reject any root the real situation forbids and finish by answering precisely what was asked.

Question 5: Linear Equations, Cost Functions and the Straight Line (20 Marks)

a) Solve the linear equation $y = 9x + 81$. b) A company has fixed costs of $9,000 for plant and equipment and variable costs of $600 for each unit of output. What is the total cost at varying levels of output? c) Find the equation of the straight line that has slope $m = 3$ and passes through the point $(-4, -6)$.

Introduction

The straight line is the simplest relationship in business mathematics and the one used most often. Its equation, $y = mx + c$, contains the whole story in two numbers: the slope m, which tells how fast y changes when x increases by one unit, and the intercept c, which tells the value

of y when x is zero. This question examines the straight line from its three everyday angles. Part (a) hands us a completed equation, y = 9x + 81, and asks us to 'solve' it — in the language of this course that means extracting the information the equation carries: where it crosses the axes, what values it pairs together, and what its graph looks like. Part (b) runs the process in the business direction: a verbal description of a cost structure — a fixed cost that must be paid even at zero output and a variable cost incurred for every unit produced — is to be turned into a total cost function and evaluated at varying levels of output, which is the daily arithmetic of budgeting and break-even thinking. Part (c) runs it in the construction direction: given a slope and a single point, build the equation of the line. A student who can move freely among words, equation, table and graph for the same line understands linear functions properly, and that four-way fluency is exactly what the three parts together are testing. Each part below is therefore presented in more than one form, with the graph of Part (a) and the cost line of Part (b) drawn in Figures 1 and 2.

Part (a): Solving y = 9x + 81 — Intercepts, Table and Graph

The equation is already in slope–intercept form, y = mx + c, with slope m = 9 and y-intercept c = 81. The slope 9 means that every increase of 1 in x increases y by 9; the intercept 81 means the line crosses the y-axis at the point (0, 81). To 'solve' the equation in the sense of this course is to find the pairs (x, y) that satisfy it and in particular the two intercepts, from which the line can be drawn. The y-intercept is found by setting x = 0: y = 9(0) + 81 = 81, giving the point (0, 81). The x-intercept is found by setting y = 0 and solving the resulting first-degree equation: 0 = 9x + 81 gives 9x = −81 and x = −9, giving the point (−9, 0). Two points determine a straight line, so the graph is the line drawn through (−9, 0) and (0, 81), shown in Figure 1. A short table of values

fills in the picture and provides checks: when x = −9, y = 0; when x = −5, y = 9(−5) + 81 = 36; when x = −2, y = 63; when x = 0, y = 81; when x = 1, y = 90; and when x = 4, y = 117. Each step of 1 in x adds exactly 9 to y throughout the table, confirming the slope, and reading the table backwards confirms that the equation can also be solved for x, x = (y − 81)/9, so that any required y can be traced to the x that produces it. If the equation is read as the demand- or supply-style relationship of business mathematics, the intercepts are the meaningful landmarks: (−9, 0) is where the quantity y falls to zero, and (0, 81) is the starting value before x has any effect at all.

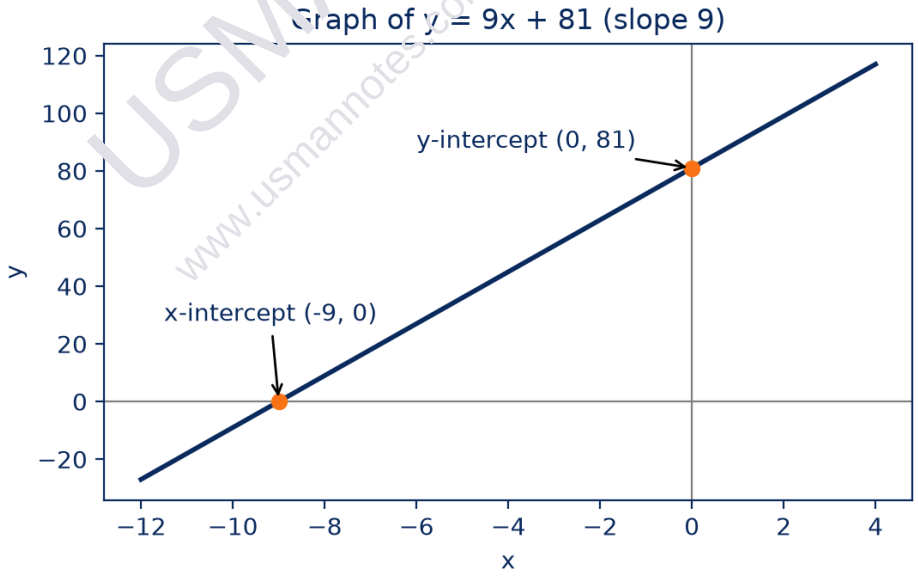


Figure 1: Graph of y = 9x + 81, crossing the axes at (−9, 0) and (0, 81).

Table 1: Values satisfying y = 9x + 81

x	−9	−5	−2	0	1	4
y = 9x + 81	0	36	63	81	90	117

Part (b): Total Cost with Fixed Cost $9,000 and Variable Cost $600 per Unit

Total cost has two ingredients. The fixed cost of $9,000 for plant and equipment is payable whatever the output — indeed it is payable at zero output, when the factory stands idle — while the variable cost of $600 per unit grows in direct proportion to output. If x units are produced, the variable cost is 600x dollars, and total cost is therefore C(x) = 9,000 + 600x. This is itself a straight-line equation: the fixed cost is the intercept and the variable cost per unit is the slope, which is why Part (b) belongs in a question on linear equations. Evaluating at varying levels of output: at x = 0, C = $9,000; at x = 5, C = 9,000 + 3,000 = $12,000; at x = 10, C = $15,000; at x = 20, C = $21,000; at x = 50, C = $39,000; at x = 100, C = $69,000; and at x = 200, C = $129,000, as set out in Table 2 and Figure 2. Every additional unit adds exactly $600 to total cost, whatever the level of output — that constancy is the meaning of the slope and the defining feature of a linear cost function. The table also lets us compute the average cost per unit, C(x)/x = 9,000/x + 600, which falls as output rises — $1,500 per unit at 10 units, $690 at 100 units, $645 at 200 units — because the fixed cost is spread over more and more units while the variable part per unit stays at $600. This spreading of fixed cost is the elementary reason firms seek larger volumes, and it is also why the model must be used with sense: no real plant produces unlimited output at an unchanged $600 per unit, so the function describes cost faithfully over the plant's normal operating range, not for ever. Within that range, the function answers the manager's three standard questions instantly — what a planned output will cost, what output a given budget permits (solve 9,000 + 600x = budget for x), and what each extra unit adds ($600).

Table 2: Total cost C(x) = 9,000 + 600x at varying levels of output

Output x (units)	0	5	10	20	50	100	200
Variable cost 600x ($)	0	3,000	6,000	12,000	30,000	60,000	120,000
Total cost C(x) ($)	9,000	12,000	15,000	21,000	39,000	69,000	129,000
Average cost per unit ($)	—	2,400	1,500	1,050	780	690	645

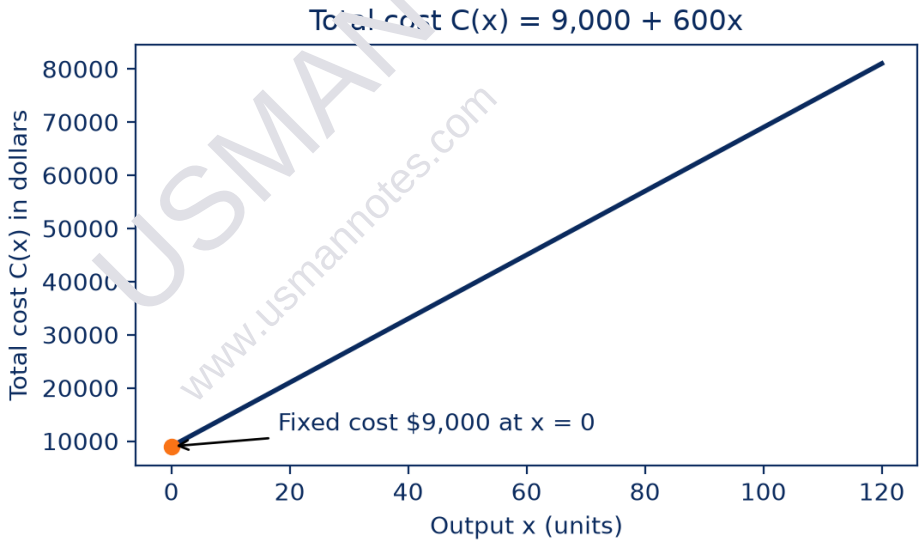


Figure 2: The total cost line C(x) = 9,000 + 600x; the intercept is the fixed cost and the slope is the variable cost per unit.

Part (c): The Line with Slope 3 through (−4, −6)

The point–slope form of a straight line builds the equation directly from a slope m and a point (x_1, y_1): $y - y_1 = m(x - x_1)$. With m = 3 and the point (−4, −6), so that $x_1 = -4$ and $y_1 = -6$, substitution gives $y - (-6) = 3(x - (-4))$, that is, $y + 6 = 3(x + 4)$. Expanding the right side, $y + 6 = 3x + 12$, and subtracting 6 from both sides gives the required equation in slope–intercept form:

y = 3x + 6. The result can be checked three ways. First, the slope of y = 3x + 6 is 3, as required ✓. Second, the point must satisfy the equation: at x = −4, y = 3(−4) + 6 = −12 + 6 = −6 ✓, so the line does pass through (−4, −6). Third, the intercepts are sensible landmarks: the line crosses the y-axis at (0, 6) and the x-axis where 3x + 6 = 0, that is, at (−2, 0). The step in which students most often err is the double negative in $y - y_1$ and $x - x_1$ when the given point has negative coordinates: $y_1 = -6$ means the formula begins $y - (-6) = y + 6$, and writing $y - 6$ instead produces the different and wrong line y = 3x + 18, which the point check immediately exposes, since 3(−4) + 18 = 6 ≠ −6. This is one more demonstration of the principle running through this whole assignment: a built-in check — here, 'the given point must satisfy the final equation' — costs a few seconds and guarantees the answer.

Conclusion

In conclusion: the line y = 9x + 81 has slope 9, y-intercept (0, 81) and x-intercept (−9, 0), and its solutions form the table and graph presented above. The company's cost structure is expressed by the linear function C(x) = 9,000 + 600x, giving total costs of $9,000 at zero output, $15,000 at 10 units, $21,000 at 20 units, $39,000 at 50 units, $69,000 at 100 units and $129,000 at 200 units, with every extra unit adding a constant $600 and average cost falling as the fixed cost is spread more thinly. The line of slope 3 through (−4, −6) is y = 3x + 6, verified by the given point itself. The unifying idea of the question is that y = mx + c is not merely a formula to memorise but a compact description — intercept for the starting value, slope for the rate of change — which can be read off an equation, built from data, costed in a table and drawn as a graph, and the business student who can pass among these four forms without hesitation has mastered

the linear model on which so much of the rest of this course, from break-even analysis onward, is built.

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