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Name:

Registration No.:

Course Code: 1349

Program: HSSC

Semester: Autumn 2026

Allama Iqbal Open University, Islamabad

Department of Mathematics

Course: Introduction to Business Mathematics (1349)

Level: HSSC

Semester: Autumn 2026

Assignment No. 01

Units: 1–5

Total Marks: 100

Pass Marks: 40

SOLVED ASSIGNMENT GUIDE — For Study & Reference

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Question No. 1

Question 1 (20 marks): (a) Define "ratio" and discuss how it is different from a rate. Convert the given ratio 40:480 into its simplest form. (b) Calculate the continued ratio (a : b : c) when (a : b = 2 : 3) and (b : c = 12 : 4). (c) Convert 32.5% into a fraction and a decimal. If a student scores 94% in 150 exams, how many marks did they secure?

Introduction

Ratio, rate and percentage are three closely related ways of comparing quantities, and together they form the language in which a very large part of business arithmetic is written. Prices are compared as ratios, wages and interest are quoted as rates, and examination results, discounts and profits are reported as percentages. A student who understands exactly what each of these tools measures, and how they differ, can move confidently through the rest of this course, because the same ideas return in discount, interest, annuities and equations. This answer defines ratio carefully, separates it from rate with the help of a comparison table, simplifies the given ratio 40:480, joins two given ratios into one continued ratio, and finally converts a percentage into a fraction and a decimal and applies a percentage to a total. Every calculation is shown step by step and checked, so that the method, and not merely the final answer, can be reproduced in an examination.

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(a) What a Ratio Is

A ratio is a comparison of two quantities of the same kind, expressed by dividing the first quantity by the second. If two quantities are a and b, their ratio is written a : b, read as “a to b”, and it has exactly the same value as the fraction a/b. The first term, a, is called the antecedent and the second term, b, is called the consequent. Because a ratio is a division, it answers the question: how many times is one quantity contained in the other, or what part of the second quantity is the first? For example, if a class contains 12 boys and 18 girls, the ratio of boys to girls is 12 : 18, which is the fraction 12/18, that is, two-thirds: there are two boys for every three girls. Three features of a ratio must always be remembered. First, the two quantities must be of the same kind and measured in the same unit before they are compared: the ratio of 2 metres to 50 centimetres is not 2 : 50 but 200 : 50, that is 4 : 1, because both lengths must first be expressed in centimetres. Second, a ratio has no unit of its own; it is a pure number, because the common unit

cancels in the division. Third, the order of the terms matters: 3 : 5 is not the same comparison as 5 : 3, since the first says the first quantity is three-fifths of the second, while the second says it is five-thirds of it. Ratios may compare more than two quantities, as in a continued ratio a : b : c, which part (b) of this question uses.

A ratio can be simplified exactly as a fraction is simplified: both terms are divided by their greatest common divisor (GCD), and the ratio is in its simplest form when the terms have no common factor except 1. For the given ratio 40 : 480, the GCD of 40 and 480 is 40, because 480 = 40 × 12. Dividing both terms by 40 gives 40 ÷ 40 = 1 and 480 ÷ 40 = 12, so 40 : 480 = 1 : 12. The check is immediate: 1 : 12 means the first quantity is one-twelfth of the second, and indeed 40 is one-twelfth of 480. The same result appears if the ratio is written as the fraction 40/480 and cancelled in stages: divide by 10 to get 4/48, then by 4 to get 1/12. In business life such a result might describe, for example, a mixture in which 40 grams of one ingredient are combined with 480 grams of another: whatever the size of the batch, the proportions stay 1 part to 12 parts, which is why the simplified form, and not the original large numbers, is the useful statement of the relationship.

How a Ratio Differs from a Rate

A rate is a comparison of two quantities of different kinds, and this single difference separates the two ideas. When a car travels 240 kilometres in 4 hours, the comparison 240 km / 4 h involves two different kinds of quantity, length and time, and its value, 60 kilometres per hour, is a rate: it carries a compound unit, kilometres per hour, which states how much of the first quantity corresponds to one unit of the second. Wages of Rs 1,500 per day, interest of 12% per annum, a price of Rs 250 per kilogram and a speed of 60 km/h are all rates. A ratio, by contrast, compares quantities of the same kind, carries no unit, and does not single out “one unit” of

anything as its reference: it simply states the relative sizes of the two quantities. There is also a useful connection between the ideas: a rate can be seen as a ratio in which the second quantity has been reduced to exactly one unit, since 240 km in 4 hours written as a ratio-like comparison 240 : 4 becomes the rate 60 : 1, that is 60 km per hour. The table below summarises the differences that an examiner looks for.

Feature	Ratio	Rate
Quantities compared	Two quantities of the same kind	Two quantities of different kinds
Unit	No unit; a pure number	A compound unit, e.g. km per hour, Rs per day
Written as	a : b, equal to the fraction a/b	Amount per one unit of the second quantity
Example	Boys : girls = 12 : 18 = 2 : 3	Speed = 240 km in 4 hours = 60 km/h
Question answered	How big is one quantity compared with the other?	How much of the first goes with one unit of the second?

(b) The Continued Ratio

A continued ratio joins three or more quantities in a single comparison, a : b : c. It can be built from two separate ratios that share a common quantity. Here we are given a : b = 2 : 3 and b : c = 12 : 4, and the shared quantity is b. The difficulty is that b appears as 3 in the first ratio and as 12 in the second, and the two statements can only be combined when b is represented by the same number in both. The standard device is to use the least common multiple of the two values of b. The LCM of 3 and 12 is 12. The first ratio must therefore be scaled so that its b-term becomes 12: multiply both terms of a : b = 2 : 3 by 4, giving a : b = 8 : 12. The second ratio already has b

= 12, so it stands unchanged as b : c = 12 : 4. Now the two ratios agree on b, and they can be written as one continued ratio: a : b : c = 8 : 12 : 4. Finally, this continued ratio can itself be simplified, because all three terms share the common factor 4. Dividing by 4 gives a : b : c = 2 : 3 : 1. Both forms are correct and equivalent; 8 : 12 : 4 shows the joining step, and 2 : 3 : 1 is the simplest form. The verification is easy and should always be done: from 2 : 3 : 1, the pair a : b = 2 : 3 as required, and the pair b : c = 3 : 1, which is exactly 12 : 4 after division by 4. A concrete picture helps: if c is one share of, say, Rs 10,000, then b takes three shares, Rs 30,000, and a takes two shares, Rs 20,000; the pairwise ratios of these amounts are precisely the ones printed in the question.

(c) Percentage, Fraction and Decimal

A percentage is a ratio whose second term is fixed at 100: "percent" means "per hundred", so 32.5% is the fraction 32.5/100. To write it as an ordinary fraction, first remove the decimal by multiplying numerator and denominator by 10, giving 325/1000, and then cancel by the GCD of 325 and 1000, which is 25: 325 ÷ 25 = 13 and 1000 ÷ 25 = 40, so 32.5% = 13/40. To write it as a decimal, divide by 100, which simply moves the decimal point two places to the left: 32.5% = 0.325. The two answers agree, since 13 ÷ 40 = 0.325, and either form can be used in calculation. Percentages, fractions and decimals are three costumes for one number, and fluency in changing between them is essential in business mathematics.

Transparency note on the printed wording: part (c) is printed in the booklet in exactly these words — "If a student scores 94% in 150 exams, how many marks did they secure?" Taken literally, "150 exams" names a count of examinations and gives no total marks, so it could not be answered. The evident intention, and the standard form of this textbook exercise, is that the examination carries a total of 150 marks, and the student scores 94% of that total. On that

reading the calculation is: marks secured = 94/100 × 150 = 0.94 × 150 = 141 marks. The check is that 141/150 = 0.94 = 94%, exactly as required. Students are advised to mention this reading in their own answer, and may also confirm it with their tutor, rather than silently changing the question.

Conclusion

A ratio compares two quantities of the same kind and has no unit; a rate compares quantities of different kinds and is expressed per unit of the second quantity. The given ratio simplifies as 40 : 480 = 1 : 12. Joining the given ratios through the common term b, scaled to the LCM 12, gives the continued ratio a : b : c = 8 : 12 : 4, that is, in simplest form, 2 : 3 : 1. Finally, 32.5% equals the fraction 13/40 and the decimal 0.325, and 94% of a 150-mark total is 141 marks. Each result has been checked against the original data, which is the habit that keeps business calculations trustworthy.

Question No. 2

Question 2 (20 marks): (a) Define discount and explain its types, and calculate the discounted price of an item originally priced at Rs 20,000 with a discount of Rs 800. (b) If a person buys a book rack for Rs 18,000 with a discount of 12%, what was the original price? (c) Calculate the profit percentage if a person bought a T.V. for Rs 5,000 and sold it for Rs 6,250.

Introduction

Discount and profit are the everyday arithmetic of trade. A shopkeeper marks goods at one price, sells them at another, and the difference between the marked price and the selling price is the discount, while the difference between the selling price and the cost price is the profit or loss. These two differences look similar but are measured against different bases, and confusing the

bases is the most common error in commercial arithmetic: a discount rate is always reckoned on the marked (original or list) price, whereas a profit percentage is always reckoned on the cost price. This answer defines discount, explains its main types, works the two discount calculations set in the question — one in which a fixed amount of discount is given, and one in which the discounted price and the discount rate are given and the original price must be recovered — and finishes with the profit percentage on the television transaction, checking each result by reversing the calculation.

(a) Discount: Definition and Types

A discount is a reduction allowed by a seller from the marked price (also called the list price or catalogue price) of an article, so that the buyer pays less than the price printed or quoted. If MP stands for the marked price and SP for the selling price, then Discount = MP − SP, and the discount percent is Discount/MP × 100, because, as noted above, the marked price is the base. Sellers offer discounts for sound business reasons: to attract customers and increase the volume of sales, to clear old or seasonal stock, to reward prompt or bulk payment, to compete with rival sellers, and to build goodwill and repeat business. A discount is therefore not a loss accepted carelessly; it is a planned sacrifice of a small part of the price in exchange for a faster, larger or safer sale. The discounted price in part (a) below is computed after the types are explained.

The main types of discount used in trade are these. (1) Trade discount is the reduction a manufacturer or wholesaler allows to a retailer or distributor off the list price; it is not recorded as a separate item in the books, the purchase simply being entered at the net price. (2) Cash discount is allowed by a seller to a buyer who pays earlier than the agreed date, for example “2/10, net 30”, meaning 2% off if payment is made within 10 days, otherwise the full amount is due in 30 days; unlike trade discount it is recorded in the accounts. (3) Quantity or bulk discount

rewards a buyer who purchases in large quantities, because big orders reduce the seller's packing, delivery and paperwork costs per unit. (4) Seasonal discount is offered at the end of a season — winter clothing in February, for example — to convert stock into cash instead of storing it. (5) Promotional discount is a short advertised reduction, such as a sale or festival offer, meant to draw customers into the shop. These types are compared in the table that follows this discussion.

Type of discount	Allowed by / to	Purpose
Trade discount	Manufacturer or wholesaler to retailer, off the list price	To leave the retailer a margin for resale
Cash discount	Seller to buyer who pays before the due date	To encourage prompt payment and improve cash flow
Quantity discount	Seller to a bulk buyer	To reward large orders and cut handling cost per unit
Seasonal discount	Seller to customers at the end of a season	To clear seasonal stock instead of storing it
Promotional discount	Seller to all customers for a short advertised period	To attract customers and increase sales volume

Worked Calculation (a): A Fixed Amount of Discount

Here the original (marked) price is Rs 20,000 and the discount allowed is Rs 800. The discounted (selling) price is found by a single subtraction: SP = MP − Discount = 20,000 − 800 = Rs 19,200. The buyer therefore pays Rs 19,200 for the item. It is instructive, although the question does not demand it, to express this discount as a percentage, because percentage discounts are how offers are usually advertised: Discount % = 800/20,000 × 100 = 4%. So the same transaction can be described in two equivalent ways — a discount of Rs 800, or a discount of 4% — and a careful

student should be able to pass from one description to the other without hesitation. The check on the price is immediate: the discount plus the price paid rebuilds the original price, 800 + 19,200 = 20,000, exactly the figure printed in the question.

(b) Recovering the Original Price from a Percentage Discount

Part (b) reverses the direction of the calculation, and it is the part of the question most often answered wrongly. The person buys the book rack for Rs 18,000 after a discount of 12%. The natural but incorrect instinct is to add 12% of 18,000 to 18,000; that is wrong because the 12% was taken off the original price, not off Rs 18,000, and a percentage can only be undone by working on the same base from which it was taken. Let the original (marked) price be P. A discount of 12% means the buyer pays the remaining 88% of P, so 0.88 × P = 18,000, and therefore P = 18,000 / 0.88 = Rs 20,454.55 approximately (exactly Rs 20,454.5454…, since 18,000/0.88 = 2,045,454.54/100 in fraction form 225,000/11). The discount actually received was 20,454.55 − 18,000 = Rs 2,454.55. The verification confirms the method: 12% of Rs 20,454.55 is Rs 2,454.55, and subtracting it gives Rs 18,000, the price printed in the question. The general rule deserves to be memorised: if a price after a discount of r% is known, the original price is the known price divided by (1 − r/100); division, not addition of a percentage, reverses a percentage reduction. The same rule, turned around, recovers a price before a percentage increase, where one divides by (1 + r/100).

(c) Profit Percentage on the Television

Profit is the excess of the selling price over the cost price, and the profit percentage expresses that excess as a percentage of the cost price, which is always the base for profit and loss calculations. Here the cost price of the T.V. is Rs 5,000 and the selling price is Rs 6,250. The profit in rupees is SP − CP = 6,250 − 5,000 = Rs 1,250. The profit percentage is therefore

Profit/CP × 100 = 1,250/5,000 × 100 = 25%. So the seller made a profit of Rs 1,250, which is a profit of 25% on cost. A quick check by the unitary method agrees: 1% of the cost price is Rs 50, and Rs 1,250 contains 1,250/50 = 25 such units, that is, 25%. It is worth adding why the base matters so much: had the same Rs 1,250 been (wrongly) expressed as a percentage of the selling price, the result would have been 1,250/6,250 × 100 = 20%, a different and incorrect answer. That 20% figure is in fact the profit margin on the selling price, a concept used in some branches of accounting, but at this level, and in the convention of this course, profit percent is reckoned on the cost price, so 25% is the correct answer to the question as set.

Conclusion

A discount is a reduction from the marked price, given in trade, cash, quantity, seasonal and promotional forms; its rate is always reckoned on the marked price. On an original price of Rs 20,000 a discount of Rs 800 (that is, 4%) leaves a discounted price of Rs 19,200. Working backwards, paying Rs 18,000 after a 12% discount means an original price of about Rs 20,454.55, found by dividing by 0.88, not by adding 12% to the amount paid. Profit percent, by contrast, is reckoned on the cost price, so buying a T.V. for Rs 5,000 and selling it for Rs 6,250 earns Rs 1,250, a profit of 25%. Remembering which base belongs to which calculation is the whole secret of this topic.

Question No. 3

Question 3 (20 marks): (a) Define simple interest and provide the formula for its calculation. If a principal amount of Rs 60,000 is borrowed at an interest rate of 12% per annum for 3 years, calculate the total interest. (b) Explain the difference between simple interest and compound

interest. Calculate the compound amount for a principal of Rs 9,000 at an interest rate of 9% compounded quarterly for one year.

Introduction

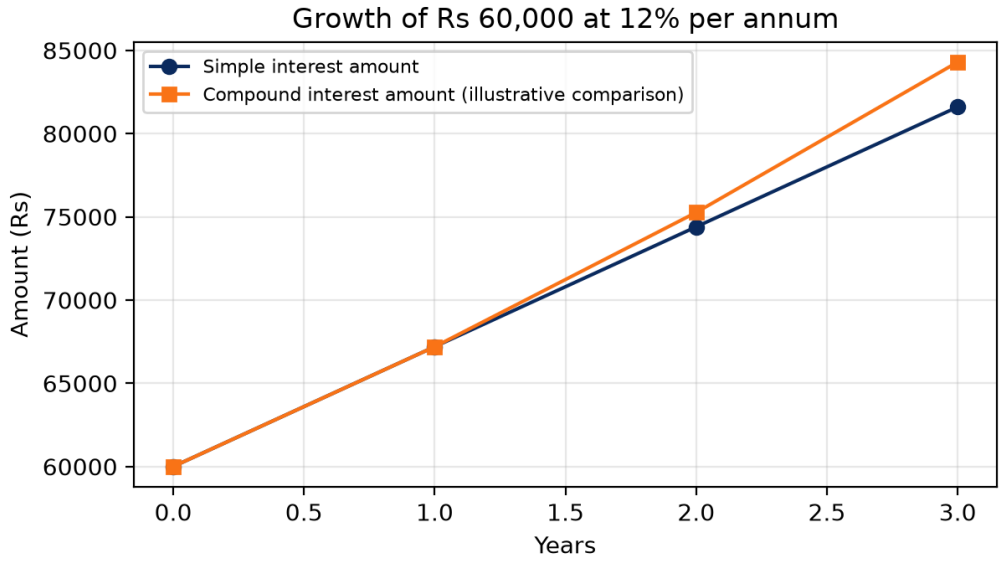
Interest is the price paid for the use of money. A borrower pays it, a depositor earns it, and in both cases it is calculated from three ingredients: the principal, the original sum; the rate, the percentage charged or earned per year; and the time for which the money is used. The two great systems for turning those ingredients into an amount of interest are simple interest, in which interest is charged only on the original principal, and compound interest, in which interest is added to the principal at regular intervals and itself begins to earn interest. The difference between the systems is small over a short period and very large over a long one, which is why the distinction matters to every saver and borrower. This answer defines simple interest, states and uses its formula on the loan of Rs 60,000, explains the differences between the two systems in a structured comparison, and then works the compound amount problem in which Rs 9,000 grows for one year at 9% with interest compounded quarterly, that is, every three months.

(a) Simple Interest and Its Formula

Simple interest is the interest calculated on the original principal alone, at a fixed rate, for the whole period of the loan or deposit, without any interest being added to the principal during the period. Because the principal never changes, the interest earned or charged is exactly the same in every year: the amount grows in a straight line, by equal steps, which is the picture shown in the figure following this section. If P is the principal in rupees, R is the rate percent per annum, and T is the time in years, the interest is $I = P \times R \times T / 100$, usually written $I = PRT/100$. The formula is nothing more than the definition of percentage applied carefully: in one year the interest is R% of P, that is PR/100; over T years at the same yearly amount it is T times as much,

PRT/100. The total sum repaid or accumulated, called the amount, is A = P + I = P(1 + RT/100). If the time is given in months or days it must first be converted into years (months divided by 12, days divided by 365), and if the rate is quoted for a period other than a year it must be adjusted to match; keeping the units of rate and time consistent is the main practical care the formula demands.

Applying the formula to the question: P = Rs 60,000, R = 12% per annum and T = 3 years. The total interest is I = 60,000 × 12 × 3 / 100. Computing in steps, the interest for one year is 60,000 × 12/100 = Rs 7,200, and over three years the interest is 7,200 × 3 = Rs 21,600, which is exactly what the formula gives directly: 60,000 × 36/100 = 21,600. The total interest is therefore Rs 21,600, and the amount to be repaid at the end of the three years is 60,000 + 21,600 = Rs 81,600. The year-by-year view — Rs 7,200 in the first year, Rs 7,200 in the second and Rs 7,200 in the third — displays the defining character of simple interest: equal interest in equal periods, because the base, the original principal of Rs 60,000, never grows.



(b) Simple Interest versus Compound Interest

Compound interest differs from simple interest in one decisive respect: at the end of each agreed interval, called a conversion or compounding period, the interest earned in that interval is added to the principal, and the next interval's interest is calculated on this enlarged sum. Interest therefore earns interest, and the amount grows not in a straight line but in a curve that becomes steeper with time, as the figure illustrates for the same principal and rate. With annual compounding the amount after n years is A = P(1 + R/100)^n. If interest is compounded m times a year, each period's rate is R/m percent and there are m × n periods, so A = P(1 + R/(100m))^(mn). The main contrasts are these: the base for simple interest is fixed, for compound interest it grows each period; simple interest is the same each year, compound interest increases each year; over one period the two are equal, but over several periods compound interest is always greater at the same rate; simple interest suits short loans and is easy to compute, while compound interest is the system actually used by banks for deposits and by lenders for most longer loans, precisely because it reflects the real opportunity value of the interest as it accumulates. The table following this section sets the comparison out formally.

Feature	Simple interest	Compound interest
Base of calculation	Original principal only, throughout	Principal plus interest already credited
Interest each period	The same in every period	Increases every period
Growth of amount	Straight line (linear)	Curve (exponential), steepening with time
Formula for amount	A = P(1 + RT/100)	A = P(1 + R/(100m))^(mn) for m compoundings a year
Typical use	Short-term loans and simple	Bank deposits and most longer

	borrowings	loans

Worked Calculation (b): Compounded Quarterly

Here P = Rs 9,000, the nominal rate is 9% per annum, the time is 1 year, and interest is compounded quarterly, so m = 4 conversions in the year. The rate per quarter is 9/4 = 2.25%, that is 0.0225 as a decimal, and the number of periods is 4 × 1 = 4. The compound amount is A = 9,000 × (1.0225)^4. Working period by period shows the mechanism and provides the check: after the first quarter the amount is 9,000 × 1.0225 = Rs 9,202.50; after the second quarter, 9,202.50 × 1.0225 = Rs 9,409.56 (rounded to the nearest paisa); after the third quarter, 9,409.56 × 1.0225 = Rs 9,621.27; and after the fourth quarter, 9,621.27 × 1.0225 = Rs 9,837.75. The compound amount is therefore approximately Rs 9,837.75, and the compound interest earned is 9,837.75 − 9,000 = Rs 837.75. A comparison underlines the lesson of the question: had the same Rs 9,000 earned simple interest at 9% for one year, the interest would have been only 9,000 × 9/100 = Rs 810, giving an amount of Rs 9,810. Compounding quarterly adds a further Rs 27.75, which is exactly the interest earned by the interest credited in the earlier quarters. The gain is modest in one year, but the same mechanism over ten or twenty years produces a very large difference, which is why the frequency of compounding is always stated in any serious loan or deposit agreement.

Conclusion

Simple interest, I = PRT/100, is charged on the original principal only and so grows in equal yearly steps: on Rs 60,000 at 12% for 3 years it comes to Rs 21,600, with an amount of Rs 81,600. Compound interest adds each period's interest to the principal, so the base grows and the amount follows A = P(1 + R/(100m))^(mn). For Rs 9,000 at 9% compounded quarterly for one

year the amount is about Rs 9,837.75, an interest of Rs 837.75, slightly more than the Rs 810 that simple interest would give. The formulas, and the habit of checking them period by period, are the foundation for the annuity calculations that follow in this course.

Question No. 4

Question 4 (20 marks): (a) Define annuity and explain its types, and calculate the future value of an ordinary annuity of Rs 1,500 payable at the end of each year for 12 years at an interest rate of 6%. (b) Find the two consecutive odd integers whose sum is 40. (c) Solve the quadratic equation using the factorization method. ($x^2 + 2x - 3 = 0$).

Introduction

This question brings together two different strands of business mathematics. Its first and largest part concerns annuities, the series of equal payments made at equal intervals that lie behind pensions, insurance premiums, loan instalments and savings plans; a formula is developed and then used to find the accumulated future value of yearly payments of Rs 1,500 over 12 years at 6%. The remaining parts change the subject to elementary algebra, the second great strand of the course: part (b) asks for two consecutive odd integers with a given sum, a standard exercise in translating words into an equation, and part (c) requires a quadratic equation to be solved by factorisation, the quickest of the three standard methods when a quadratic splits into neat factors. Each part is answered in full, with the reasoning stated before the arithmetic, because in both finance and algebra the marks lie in the method: a correct set-up earns most of the credit even if a slip occurs later, and it is the set-up that this answer models carefully.

(a) Annuity: Definition and Types

An annuity is a series of equal payments (or receipts) made at equal intervals of time over a fixed period. The amount of each payment is often denoted by R (or PMT), the length of the whole series is its term, and the interval between payments — a year, a half-year, a quarter or a month — is the payment period. Familiar examples are a yearly pension, a monthly house-rent instalment paid under an agreement, an insurance premium paid every year, and the equal monthly payment that repays a loan. Two values are attached to every annuity: its future value, the total to which the payments accumulate with interest by the end of the term, and its present value, the single sum today that is equivalent to the whole series; part (a) asks for a future value, and the present value of a related series appears in Question 5. The classification of annuities most often used, and set out in the table, is by the timing of the payments and by whether the term is fixed: an ordinary annuity (also called an annuity immediate) has its payments at the end of each period; an annuity due has them at the beginning of each period, so each payment earns one extra period of interest; a certain (or fixed-term) annuity runs for a stated number of periods regardless of any person's life; a contingent annuity, like a life pension, continues only while an event such as survival lasts; and a perpetuity is an annuity whose payments continue for ever, as with some endowments and preference shares.

The payments of an ordinary annuity do not all earn interest for the same length of time, and the future value formula is best understood by seeing why. The last payment, made at the end of the twelfth year, earns no interest at all; the eleventh payment earns one year of interest; the tenth earns two years, and so on back to the first payment, which earns eleven years of interest. The future value is therefore R(1 + i)^11 + R(1 + i)^10 + … + R(1 + i) + R, a geometric series with ratio (1 + i), whose sum is the standard formula FV = R × [((1 + i)^n − 1) / i], where i is the interest rate per period as a decimal and n is the number of payments. Substituting the figures of

the question, R = Rs 1,500, i = 0.06 and n = 12: first (1.06)^12 = 2.012196 (to six decimal places), so the bracket becomes (2.012196 − 1) / 0.06 = 1.012196 / 0.06 = 16.869941. Multiplying by the payment, FV = 1,500 × 16.869941 = Rs 25,304.91 approximately. The interpretation is worth stating: the saver actually pays in 12 × 1,500 = Rs 18,000, yet the fund at the end is worth Rs 25,304.91, so Rs 7,304.91 of the final amount is interest earned by the earlier payments. That gap between money paid in and value accumulated is the entire point of an annuity, and it is why starting a savings annuity early, which lengthens the time over which the early payments compound, is so powerful.

Type of annuity	When payments are made / how the term runs	Example
Ordinary annuity (annuity immediate)	At the end of each period	Yearly payment into a savings fund, valued at the term's end
Annuity due	At the beginning of each period	Rent or an insurance premium paid in advance
Annuity certain	For a fixed, stated number of periods	Loan instalments over exactly 12 years
Contingent annuity	Only while a stated event, such as survival, lasts	A life pension
Perpetuity	Payments continue for ever	Income from a permanent endowment

(b) Two Consecutive Odd Integers with Sum 40

Consecutive odd integers are odd numbers that follow one another in the odd sequence — 17, 19, 21, 23, … — and any two neighbours differ by 2. Let the first (smaller) odd integer be n. The

next odd integer is then n + 2. The condition of the question is that their sum is 40 (the number printed in the booklet, read from its equation object, is 40), so the equation is n + (n + 2) = 40, that is, 2n + 2 = 40. Subtracting 2 from both sides gives 2n = 38, and dividing by 2 gives n = 19. The two integers are therefore 19 and n + 2 = 21. The check is immediate: 19 and 21 are both odd, they are consecutive in the odd sequence, and 19 + 21 = 40, exactly the sum required. The method generalises to all problems of this family: represent the unknown by a letter, use the known gap between the numbers (1 for consecutive integers, 2 for consecutive even or consecutive odd integers) to write the next number, form an equation from the stated condition, solve it, and finally verify the answer against every condition in the words, not merely the one used to build the equation.

(c) Solving $x^2 + 2x - 3 = 0$ by Factorisation

Factorisation solves a quadratic equation by reversing the multiplication of two brackets: if the left-hand side can be written as a product of two linear factors, then the equation is satisfied whenever either factor is zero, because a product is zero only when at least one of its factors is zero. For $x^2 + 2x - 3$, we need two numbers whose product is the constant term, -3, and whose sum is the coefficient of x, namely $+2$. The factor pairs of -3 are $(1, -3)$ and $(-1, 3)$; the second pair has sum $-1 + 3 = +2$, so the required numbers are $+3$ and -1. Split the middle term accordingly: $x^2 + 3x - x - 3 = 0$. Group in pairs: $x(x + 3) - 1(x + 3) = 0$, and the common factor $(x + 3)$ gives $(x + 3)(x - 1) = 0$. Setting each factor to zero separately, $x + 3 = 0$ gives $x = -3$, and $x - 1 = 0$ gives $x = 1$. The solutions are $x = -3$ and $x = 1$. Both are verified by substitution in the original equation: for $x = 1$, $1 + 2 - 3 = 0$; for $x = -3$, $9 - 6 - 3 = 0$. Factorisation is the method of choice whenever the factors are whole numbers, because it is fast and exact; when no such

pair exists, the quadratic formula of Question 5 must be used instead, and comparing the two methods on exercises like this one shows exactly where the boundary between them lies.

Conclusion

An annuity is a series of equal payments at equal intervals; its ordinary form, with payments at the end of each period, has future value R[((1 + i)^n − 1)/i], which for Rs 1,500 a year for 12 years at 6% gives about Rs 25,304.91, of which Rs 7,304.91 is interest. The two consecutive odd integers summing to 40 are 19 and 21, found by solving n + (n + 2) = 40. The quadratic $x^2 + 2x - 3 = 0$ factorises as (x + 3)(x − 1) = 0, giving the two solutions x = −3 and x = 1, both confirmed by substitution. In each part the answer was checked against the original statement, the routine that distinguishes reliable work in both finance and algebra.

Question No. 5

Question 5 (20 marks): (a) Solve the linear equation: (5x − 10 = 3x + 6). (b) What is the present value of an annuity due of Rs 2,000 per year for 5 years at an interest rate of 5%? (c) Use the quadratic formula to solve: ($2x^2 - 4x + 1 = 0$). (d) Solve the simultaneous equations: (3x + 4y = 24) and (2x − 3y = 15).

Introduction

The final question of this assignment is a tour of the solving techniques of the course, one for each of the main equation families. Part (a) is a linear equation in one unknown, solved by collecting the unknown on one side and the numbers on the other. Part (b) returns to annuities, but asks the reverse question from Question 4: instead of accumulating payments forward to a future value, it discounts an annuity due backwards to its present value. Part (c) requires the quadratic formula, the universal method that solves any quadratic equation, including the ones,

like this one, whose answers are not whole numbers and so cannot be found by factorisation. Part (d) presents two linear equations in two unknowns that must hold simultaneously, solved here by elimination. A common discipline runs through all four parts: every solution is checked by substituting it back into the original statement, because an answer that does not satisfy the original equation is not an answer at all, however neatly it was obtained.

(a) The Linear Equation 5x − 10 = 3x + 6

A linear (first-degree) equation contains the unknown only to the first power, and it is solved by applying the same operation to both sides until the unknown stands alone; whatever is done to one side must be done to the other, so that the equality is preserved at every step. Start with 5x − 10 = 3x + 6. Subtract 3x from both sides to gather the x-terms on the left: 5x − 3x − 10 = 6, that is, 2x − 10 = 6. Add 10 to both sides to gather the numbers on the right: 2x = 16. Divide both sides by the coefficient 2: x = 8. The solution is x = 8. The check substitutes 8 into both sides of the original equation: the left side is 5(8) − 10 = 40 − 10 = 30, and the right side is 3(8) + 6 = 24 + 6 = 30. The two sides agree, so x = 8 is confirmed. The same sequence — remove brackets if any, collect unknowns on one side, collect constants on the other, divide by the coefficient, and check — solves every linear equation of this course, and stating each step in writing, as here, is what protects a candidate from sign errors, the one real danger in such questions.

(b) Present Value of an Annuity Due

The present value of an annuity is the single sum which, if invested today at the given rate, would exactly provide for all the payments of the annuity; equivalently, it is the sum of the discounted values of the individual payments, each payment being divided by (1 + i) raised to the number of periods until it is paid. For an ordinary annuity of R per period for n periods at rate i, the present value is PV = R × [(1 − (1 + i)^(−n)) / i]. An annuity due, whose payments fall at the

beginning of each period, brings every payment one period earlier than the corresponding ordinary annuity, so each payment is discounted by one period less and the whole present value is larger by exactly the factor (1 + i): PV(due) = PV(ordinary) × (1 + i). In the question, R = Rs 2,000, i = 0.05 and n = 5. First compute (1.05)^(−5): since (1.05)^5 = 1.276282, its reciprocal is 0.783526. The ordinary-annuity factor is then (1 − 0.783526) / 0.05 = 0.216474 / 0.05 = 4.329477, so the present value of the corresponding ordinary annuity would be 2,000 × 4.329477 = Rs 8,658.95. Applying the annuity-due adjustment, PV = 8,658.95 × 1.05 = Rs 9,091.90 approximately. This is the amount that must be set aside today at 5% to pay out Rs 2,000 at the start of each of the next five years. A payment-by-payment view confirms it: the five payments are worth today 2,000 (paid at once), 2,000/1.05 = 1,904.76, 2,000/1.05² = 1,814.06, 2,000/1.05³ = 1,727.68 and 2,000/1.05⁴ = 1,645.40, and these add to Rs 9,091.90. Comparing with Question 4 underlines the unity of the topic: future value pushes payments forward with interest, present value pulls them back with discounting, and the timing of the payments — end or beginning of the period — is handled by a single factor of (1 + i).

(c) The Quadratic Formula on $2x^2 - 4x + 1 = 0$

The quadratic formula states that for any equation $ax^2 + bx + c = 0$ with $a \neq 0$, the solutions are $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$. The quantity under the root, $D = b^2 - 4ac$, called the discriminant, decides the character of the answers before any arithmetic is done: if $D > 0$ there are two different real solutions, if $D = 0$ the two solutions coincide, and if $D < 0$ there are no real solutions. For $2x^2 - 4x + 1 = 0$, the coefficients are $a = 2$, $b = -4$ and $c = 1$. The discriminant is $D = (-4)^2 - 4(2)(1) = 16 - 8 = 8$, which is positive, so there are two real solutions, but since 8 is not a perfect square they will involve a square root, which is precisely why factorisation, the method of Question 4(c), cannot produce them neatly and the formula is needed. Substituting: $x = [4 \pm$

√8] / 4 = [4 ± 2√2] / 4 = 1 ± √2/2. With √2 ≈ 1.4142, the two solutions are x ≈ 1 + 0.7071 = 1.7071 and x ≈ 1 − 0.7071 = 0.2929. The check uses the two relations every pair of roots must satisfy: the sum of the roots should equal −b/a = 4/2 = 2, and indeed 1.7071 + 0.2929 = 2; the product of the roots should equal c/a = 1/2, and indeed 1.7071 × 0.2929 ≈ 0.5000. A direct substitution also works: for x = 1.7071, 2x² − 4x + 1 ≈ 5.8284 − 6.8284 + 1 = 0. Exact answers in surd form, 1 ± √2/2, should always be given alongside the decimal approximations, since the decimals are rounded but the surds are exact.

(d) The Simultaneous Equations 3x + 4y = 24 and 2x − 3y = 15

Two equations in two unknowns are called simultaneous because a solution must satisfy both at the same time: it is the pair of values (x, y) at which the two straight lines cross. The elimination method removes one unknown by making its coefficients equal in the two equations and then adding or subtracting. Label the equations (i) 3x + 4y = 24 and (ii) 2x − 3y = 15. To eliminate y, multiply (i) by 3 and (ii) by 4, so that the y-coefficients become +12 and −12: equation (iii) is 9x + 12y = 72 and equation (iv) is 8x − 12y = 60. Adding (iii) and (iv) cancels y exactly: 17x = 132, so x = 132/17 ≈ 7.7647. Substitute this into (i): 3(132/17) + 4y = 24, that is 396/17 + 4y = 24, so 4y = 24 − 396/17 = (408 − 396)/17 = 12/17, giving y = 3/17 ≈ 0.1765. The solution is therefore x = 132/17 (approximately 7.76) and y = 3/17 (approximately 0.18). The check must use both original equations: in (i), 3(132/17) + 4(3/17) = (396 + 12)/17 = 408/17 = 24, correct; in (ii), 2(132/17) − 3(3/17) = (264 − 9)/17 = 255/17 = 15, correct. The fractional answer deserves a remark, because students often assume that a non-integer result signals a mistake: it does not. The coefficients in the printed equations simply determine the fractions 132/17 and 3/17, the substitution check proves them exactly, and exact fractions, not rounded decimals, are the proper

final answer. The same elimination idea, extended, is what the matrix methods of Assignment 2 organise for larger systems.

Conclusion

Four techniques were exercised and each answer verified. The linear equation $5x - 10 = 3x + 6$ gives $x = 8$, confirmed by substitution. The present value of an annuity due of Rs 2,000 a year for 5 years at 5% is about Rs 9,091.90, the ordinary annuity value of Rs 8,658.95 increased by one period's interest factor. The quadratic formula applied to $2x^2 - 4x + 1 = 0$ gives $x = 1 \pm \sqrt{2}/2$, approximately 1.7071 and 0.2929, whose sum and product match $-b/a$ and c/a. The simultaneous equations give $x = 132/17$ and $y = 3/17$, satisfying both equations exactly. Method plus verification is the pattern to carry into the examination hall.

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